M 21126

Reg. No. : ....

Name : ....

Second Semester M.A./M.Sc./M.Com. Degree (Regular/Supplementary/ Improvement) Examination, March 2012

STATISTICS

Paper – 2.4 : Inference 1

Time : 3 Hours Max. Marks : 70

Instructions : *Answer* ***any 5*** *questions without omitting* ***any*** *Unit.*
All *questions carry* ***equal*** *marks.*

UNIT – I

I. a) Define minimal sufficient statistics. Obtain minimal sufficient statistics for the family of distributions with pdf's given below

i) $f(x, \theta) = \frac{1}{\theta} - \frac{\theta}{2} \le x \le \frac{\theta}{2}$ ii) $f(x, \theta) = \frac{1}{2} e^{-|x-\theta|} = \infty < x < \infty$

b) What do you mean by a complete family ? Examine whether the family of distributions

$$f(x, \theta) = 2\theta \text{ if } 0 < x < \frac{1}{2} \; (0 < \theta < 1)$$

$$= 2(1-\theta) \text{ if } \frac{1}{2} < x < 1 \text{ is complete.}$$

II. a) Define :

i) Unbiasdness and

ii) Consistency

Let $X_1, X_2 \ldots\ldots X_n$ be a random sample of size n from $P(\lambda)$. Discuss the unbiasdness and consistency of the estimates $T = \frac{2}{n(n+1)} \sum_{i=1}^{n} iX_i$ for λ.

b) State and prove Neyman-Fisher factorization theorem in the discrete case.

P.T.O.

UNIT – II

III. a) State and prove the invariance property of MLE. Point out its importance.

b) Define least square estimator. Let X_1, X_2 and X_3 be independent random variables with common variance σ^2 and means given by $EX_1 = \theta_1+\theta_2$, $EX_2 = \theta_1+\theta_3$ and $EX_3 = \theta_2 + \theta_3$. Obtain the least square estimates of $\theta_1+\theta_2+\theta_3$ and its variance.

IV. a) Explain the method of moments. Show that moment estimators are always consistent. Obtain moment estimators of parameters when $f(x) = \frac{1}{\lambda}e^{-\frac{1}{\lambda}(x-\theta)}$ $x > \theta$.

b) Distinguish between method of minimum chi-square and method of modified minimum chi-square procedures. Illustrate those using an example.

UNIT – III

V. a) Define Fisher information measure. Let $X_1, X_2 \ldots X_n$ be a random sample from a distribution with pmf

$$P(X = x) = p + c(-p)\,e^{-\lambda} \text{ if } x = 0$$
$$= (1-p)\frac{e^{-\lambda}\lambda^x}{x^1} \quad x = 1, 2, \ldots$$

$0 < p < 1$, $\lambda > 0$ and p and c are known. Obtain Fisher information for λ.

b) State and establish Chapmen-Robbin's inequality.

VI. a) Define UMVUE. Given a random sample of size n from $N(\mu, \sigma^2)$, find UMVUE of

i) $2\mu + 3\sigma$ and

ii) μ/σ.

b) State and establish a necessary and sufficient condition for an unbiased estimates to be UMVUE.

UNIT – IV

VII. a) Explain :

i) Prior distribution

ii) Posterior distribution and

iii) Bayes risk. Let $X \sim P(\lambda)$. The prior distribution is $\pi = (\lambda) = e^{-\lambda}\lambda > 0$. Under squared error loss function find the Bayes estimator of λ. Is it unique ?

b) Describe minimax method of estimator. Show that if the risk function is constant then Bayes and minimax estimators coincide.

VIII. a) Define Pitman estimator for location parameter. Let $f(x, \theta) = \frac{1}{2} e^{-|x-\theta|}$. Find the Pitman estimator for θ.

b) Define Bayer estimator. Let X~b(n, p). The prior of p is uniform over (0, 1). Find Bayes estimator of p^2 under squared error loss.

UNIT – V

IX. a) Explain the methods of construction of confidence intervals using central limit theorem and Chebyshev's inequality.

b) Define unbiased confidence interval. Let $X \sim N(\mu, \sigma^2)$. Obtain an unbiased $100(1-\alpha)\%$ confidence interval for μ.

i) When σ is known and

ii) When σ is unknown.

X. a) Explain shortest length confidence interval. Let $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$. Find $100(1-\alpha)\%$ shortest length confidence interval for $\left(\frac{\sigma_1^2}{\sigma_2^2}\right)$ based on two independent set of samples. Assume that x and Y are independent.

b) Distinguish between UMA and UMAU confidence intervals. Prove or disprove UMA confidence intervals are always shortest length confidence intervals.
