M 17525

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree Examination, March 2010
STATISTICS (Paper – 2.4)
Inference – I

Time : 3 Hours Max. marks : 70

*Instructions : Answer **any 5** questions without omitting **any** Unit.*
*All questions carry **equal** marks.*

UNIT – I

I. a) Define

i) Minimal sufficiency and

ii) Completeness let $X \sim N(\theta, \theta^2)$.

Find a sufficient statistic for θ. Is the family $N(\theta, \theta^2)$ complete ? Justify ?

b) Define ancillary statistic. If T is a complete sufficient statistic, prove that any ancillary statistic is independent of T. Is converse true ? Justify ?

II. a) State and prove factorization theorem on sufficiency.

b) Define :

i) unbiasedness and

ii) consistency

Let $f(x, \theta, \lambda) = \frac{1}{\lambda} e^{\frac{-1}{\lambda}(x-\theta)} \quad x > \theta$

Find consistent and unbiased estimators of θ, λ and $\theta + \lambda$.

UNIT – II

III. a) Define maximum likelihood estimator. Prove or disprove : Maximum likelihood estimaters are always unbiased.

b) Explain the method of moments. Prove that moment estimators are consistent. Find the moment estimator for μ and σ^2 when X follows lognormal distribution with parameters μ and σ^2.

P.T.O.

IV. a) Under regularity conditions to be stated show that maximum likelihood estimators are asymptotically normal.

b) Describe least square method for estimation. Let $f(x, \theta) = e^{-(x-\theta)}, x \geq \theta$ show that $T = \frac{1}{n}\sum_{1}^{n} X_i - 1$ is BLUE for θ.

UNIT – III

V. a) State Cramer - Rao lower bound. Find Cramer-Rao lower bound for an unbiased estimator when $f(x, \theta) = \frac{1}{2\theta} e^{-\frac{|x|}{\theta}}, -\infty < x < \infty, \theta > 0$. Find an unbiased estimator of θ^2 whose variance attains Cramer Rao-lower bound.

b) Define Fisher information matrix. Find Fisher informaiton matrix when $X \sim N(\mu, \sigma^2)$.

VI. a) State and prove Rao-Blackwell theorem.

b) Define uniformly minimum variance unbiased estimator.
Let $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$. Find UMVUE's of

i) $\mu_1 - \mu_2$ ii) σ_1^2 / σ_2^2.

UNIT – IV

VII.a) Define
i) loss function
ii) risk function and
iii) Baye's risk.

Let $X \sim P(\lambda)$. Find Bayes estimator of λ under squared error loss function when the prior $\pi(\lambda) = e^{-\lambda}, \lambda > 0$.

b) Define minimax estimator. Let $X_1, X_2 .... X_n$ be i.i.d with $f(x, \theta) = e^{-(x-\theta)}, x > \theta$. Let $\pi(\theta) = e^{-\theta}, \theta > 0$. Under squared error loss function show that $X_{(1)} - \frac{1}{n}$ is minimax, whose $X_{(1)} = \min(X_1, ... X_n)$.

VIII. a) Define admissible rule. Let X_1, X_2.... X_n be i.i.d with normal distribution $N(\theta, 1)$. Show that $\overline{X}$ is admissible estimator of under squared error loss function.

b) Let $X \sim b(n, \theta)$. Suppose that prior distribution of θ is U (0, 1). Find the Bayes estimator θ using loss function $L(\theta, T) = \frac{(\theta - T)^2}{\theta(1-\theta)}$. Find a minimax estimator of θ.

UNIT – V

IX. a) What do you mean by interval estimation ? What are different methods for construction of confidence intervals. Illustrate one method with a suitable example.

b) Define shortest length confidence interval. Let $f(x, \theta) = e^{-(x-\theta)}$, $x > \theta$. Find the shortest length confidence internal for θ at level $1-\alpha$, based on a sufficient statistic.

X. a) Explain unbiased confidence intervals Let $X \sim N(\mu_1, \sigma^2)$ and $Y \sim N(\mu_2, \sigma^2)$ Find 100 $(1-\alpha)$ % uniformly most accurate confidence interval for $\mu_1 - \mu_2$ based on two sets of independent samples from normal populations.

b) Define Bayesian confidence interval Let X_1, X_2, ...X_n be a sample from a geometric distribution with parameter θ. The prior density of θ is beta with parameters α and β. Find a 100 $(1-\alpha)$ % Bayesian confidence interval for θ.
