M 15937

Reg. No. : ....

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Second Semester M.Sc. Degree Examination, May 2009

STATISTICS

Paper – 2.4 : Inference – I

Time : 3 Hours

Max. Marks : 70

Instructions : *Answer any 5 questions without omitting any Unit. All questions carry equal marks.*

UNIT – I

I. a) Distinguish between minimal sufficiency and sufficiency. Give examples. Let X follow $P(\theta)$. Show that $T = \sum_{i=1}^{n} l_i X_i$ is sufficient if and only if $l_1 = l_2 = \_\_\_\_\_\_\_\_ = l_n$.

b) Define consistency. Let $f(x, \theta) = \frac{1}{\theta}, 0 < x < \theta$. Show that $X_{(n)} = \max(X_1, ...X_n)$ is consistent for θ.

II. a) State and prove Basu's theorem on sufficiency.

b) Define unbiased estimator. Prove or disprove the following :

i) Unbiased estimators are always consistent

ii) There exist an unbiased estimator for every parametric function.

UNIT – II

III. a) Define moment estimator. Let $X_1, X_2 ....X_n$ be a sample from beta distribution with parameters α and β. Find the moment estimator for (α, β). Are they consistent for (α, β).

b) Describe the method of maximum likelihood estimation. Find MLE of θ when :

i) $X \sim N(\theta, \theta), \theta > 0$

ii) $X \sim U(\theta, \theta + 1), \theta > 0$.

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IV. a) Describe the method of minimum chi-square. Prove or disprove. Minimum chi-square estimators are always consistent.

b) Under the regularity conditions, to be stated, show that the likelihood equation provides a consistent estimator and it is unique.

UNIT – III

V. a) State and prove Cramer-Rao unequality.

b) Define Fisher information matrix. Find Fisher infor matrix when X follows gamma distribution with parameters α and β.

VI. a) Define uniformly minimum variance unbiased estimator. Let X~N(θ, 1). Find UMVUE of

i) $P(X \leq U)$ and

ii) $\xi_p = F^{-1}(p)$ where $F(\cdot)$ is the distribution function.

b) Prove : A necessary and sufficient condition for an unbiased estimator T of $\psi(\theta)$ to be most efficient is that T be sufficient and the relation $T(X) - \psi(\theta) = k(\theta) \frac{\partial}{\partial\theta} \log L(\theta, x)$ for some $k(\theta)$.

UNIT – IV

VII. a) Distinguish between prior and posterior distributions. Let X~N(θ, 1). Find a conjugate prior for θ. Using this, find Bayes estimator of θ under squared error loss function.

b) Define minimax estimator. If the risk function corresponding to a Bayes estimator T is constant, show that T is minimax.

VIII. a) Define admissible and inadmissible rules. Find an admissible estimator of θ when X~N(θ, 1).

b) Define Pitman estimator for a scale parameter θ. Find Pitman estimator of θ when X~N(0, θ).

UNIT – V

IX. a) Define uniformly most accurate interval. Let $X \sim U(0, \theta)$. Find a uniformly most accurate confidence interval for θ.

b) Explain shortest length confidence interval. Let $X \sim N(0, \sigma^2)$. Find shortest length confidence interval for σ^2.

X. a) Describe large sample confidence intervals. Find $100(1-\alpha)\%$ large sample confidence interval for θ when $X \sim P(\theta)$.

b) Define uniformly most accurate unbiased confidence interval.

Let $X_1, X_2 \ldots\ldots X_n$ be i.i.d. with $f(x, \theta) = \frac{\theta}{x^2}, x \geq \theta$. Find uniformly most accurate confidence interval for θ based on the pivot $\theta / X_{(1)}$.
