M 11989

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# Second Semester M.Sc. Degree Examination, May 2006
## STATISTICS
## Paper – 2.4 : Inference – I

Time: 3 Hours — Max. Marks: 70

*Instruction: Answer any five questions, choosing **one from each unit**.*

### UNIT – 1

1. a) Define consistency. Prove or disprove 'consistent estimators are unique'. **(5+4+5=14)**

   b) What do you mean by sufficiency and minimal sufficiency ?

   c) Let $f(x;\theta,p)=(1-p)p^{x-\theta}$ $x=\theta,\ \theta H....;0<p<1$. Find a sufficient statistic, based on a sample of size n when

      i) p is unknown and $\theta$ is known

      ii) both p and $\theta$ are unknown.

2. a) Define unbiasedness. Let $X\sim b(1,\theta^2)$. Does there exist an unbiased estimate of $\theta$. **(5+5+4=14)**

   b) State and prove factorization criterion on sufficiency.

   c) Define complete family. Let X be a random variable with probability distribution

      $P_Q\{X=-1\}=\theta$ and $P_\theta\{X=x\}=(1-\theta)^2\,\theta^x, x=0,1,2\ldots$ Is $\{P_\theta:\theta\in(0,1)\}$ a complete family ?

### UNIT – 2

3. a) A random sample of size n is taken from the lognormal density

$$f(x)=\frac{1}{\sqrt{2\pi}}\sigma\frac{1}{x}e^{-\frac{1}{2\sigma^2}(\log x-\mu)^2} \qquad x>0.$$

   **(7+7=14)**

   Find the estimate of $\mu$ and $\sigma$ using method of moments.

   b) Define maximum likelihood estimator (MLE) show that MLE's are consistent and functions of sufficient statistic, if it exists.

P.T.O.

4. a) Explain the method of minimum chi-square estimation. Illustrate it with an example. **(4+5+5=14)**

b) Let $X \sim N(\mu, \sigma^2)$. When the sample size n is one, show that no MLE of $(\mu, \sigma^2)$ exists.

c) Define best linear unbiased estimator (BLUE). Prove or disprove 'Least square estimators are BLUE's.

UNIT – 3

5. a) Define minimum variance bound estimator. Find MVBE of $\theta$ when $f(x;\theta) = \frac{1}{\theta} e^{-x/\theta} \quad x > 0.$ **(5+5+4=14)**

b) Define Fisher information. Find Fisher information when

$$f(x;\theta) = \frac{1}{\pi(1+(x-\theta)^2)} \quad -\infty < x < \infty$$

c) Show that a necessary and sufficient condition for an estimate T to be UMVUE is that Cov (TU) = 0 $\forall \theta \in \Omega$ where U is such that E(U) = 0 $\forall \theta \in \Omega$.

6. a) State and prove Rao-Blackwell theorem. **(6+4+4=14)**

b) Let X be a density $f(x;\theta) = \frac{a(x)\theta^x}{g(\theta)} \; x = 0, 1, 2 \ldots \; a(x) > 0$. Based on a sample of size n, find UMVUE of $\psi(\theta) = \theta^r$ (r > 0 is an integer).

c) Prove or disprove: UMVUE's are BAN estimators.

UNIT – 4

7. a) Define: **(5+5+4=14)**

i) prior distribution

ii) Posterior distribution and

iii) loss function

b) Show that under squared error loss function, the Bayes estimate is the Posterior mean of the distribution.

c) Let X~N($\theta$,1), and the prior $\pi(\theta) \sim N(0, 1)$. Using a squared error loss function, find Bayes estimate of $\theta$. Deduce the M.L.E. of $\theta$.

8. a) Explain the method of minimax estimation. (5+5+4=14)

b) Let d* be a minimax estimate for $\theta$ with respect to the squared error loss function. Show that ad* + b is minimax for $a\theta+b$, a, b are constants.

c) Define admissibility. Check whether the estimator $\overline{X}$ is admissible or not for $\theta$ when X~b(1, $\theta$).

UNIT – 5

9. a) Explain the method of construction of confidence intervals using pivotal method. (4+5+5=14)

b) Let $X_1, X_2, \ldots X_m$ and $Y_1, Y_2, \ldots Y_n$ be i.i.d samples from $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ respectively. Find a confidence interval of $\mu_1 - \mu_2$, at level $1-\alpha$, when $\sigma$ is unknown.

c) Define UMA family of confidence interval. Let X~U(0, $\theta$). Find a UMA family of confidence interval for $\theta$ at level $1-\alpha$.

10. a) Define unbiased confidence interval. Let X has the density

$$f(x;\theta) = \frac{1}{\theta} e^{-x/\theta} \quad x > 0.$$

Find an unbiased confidence interval based on $\frac{2\sum_{1}^{n} X0}{\theta}$. (5+5+4=14)

b) Let X~b(1, $\theta$). Find the shortest confidence interval for $\theta$ at level $1-\alpha$, based on a sufficient statistic.

c) Explain the concept of fiducial interval.

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