Reg.No.....................

Name.....................

M 10587

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY 2005

STATISTICS

INFERENCE – I

(2004 Admns.)

Time: Three Hours

Maximum: 70 Marks

*Answer any **five** questions, choosing one from each unit.*

UNIT - I

1. a) Let $x_1, x_2 \ldots x_n$ be i i d observations from U (0, θ). Show that $T(x_1, x_2, \ldots x_n) = \left(\prod_{i=1}^{n} x_i\right)^{1/n}$ is consistent for θ/e.

   b) Define sufficient statistics. If T is sufficient and T = g(U), then show that U is sufficient.

   c) Let $x_1, x_2, \ldots x_n$ be a sample from U (θ - ½ , θ + ½) θ ∈ R. Show that $T(x_1, x_2, \ldots x_n) = (\min x_i, \max x_i)$ is sufficient for θ but not complete.

   (5 + 5 + 4 = 14)

2. a) Prove or disprove : 'Unbiased estimators are consistent'.

   b) Let x_1 and x_2 be two observations from a population with density

   $$f(x;\theta) = \frac{1}{\pi}\,\frac{1}{1+(x-\theta)^2} \qquad -\alpha < x < \alpha,\ -\alpha < \theta < \alpha$$

   Show that $T = (X_{(1)}, X_{(2)})$ is minimal sufficient for θ.

   c) Let $x_1, x_2 \ldots x_n$ be a sample from

   $$f_N(x) = \begin{cases} 1/N & x = 1,2,\ldots N \\ 0 & \text{otherwise} \end{cases}$$

   Show that the family P = { PN : N ≥ 2 } is not complete.

   (4 + 5 + 5 = 14)

UNIT- II

3. a) Define maximum likelihood estimator (M L E)

   Prove or disprove : M L E's are unique.

   b) Let x1, x2, ... Xn be a sample from the density

   $$f(x) = \frac{m^p}{\sqrt{p}}\, e^{-mx}\, x^{p-1} \qquad x > 0$$

   Obtain method of moment estimates for m and p.

   c) Describe the method of least square estimation.

   (4 + 5 + 5 = 14)

Contd......2

4. a) Under the conditions to be stated, prove that the likelihood equation ha[s] consistent solution $\hat{\theta}n$. Also show that $\hat{\theta}n$ is asymptotically normal with mean θ [a] variance $\frac{1}{n\, I_x(\theta)}$.

b) Explain the method of minimum variance estimation.

(8 + 6 = 14)

UNIT – III

5. a) State Cramer-Rao inequality. Obtain the Cramer-Rao lower bound for the variance of an unbiased estimator θ, when

$$f(x; \theta) = \theta(1-\theta)^x, \quad x = 0,1,2\ldots \quad 0 < \theta < 1$$

b) Find Fisher information matrix of μ and σ when $X \sim N(\mu, \sigma^2)$.

c) Define CAN estimator. Prove or disprove. Minimum variance bound estimators are CAN estimators.

(5 + 4 + 5 = 14)

6. a) State and Prove Lehman – Scheffe Theorem

b) Let $X \sim N(\theta, 1)$. Find uniformly minimum variance unbiased estimator for $F(u) = P(X < U)$.

c) Show that a necessary and sufficient condition for an estimate T of θ to be most efficient is that T be sufficient and the density can be written in the form

$$\frac{\delta \log f(x; \theta)}{\delta\theta} = K(\theta)\,[T(X) - \theta].$$

(6 + 4 + 4 = 14)

UNIT – IV

7. a) Explain (i) loss function and (ii) Risk function, with examples.

b) Let $X \sim P(\lambda)$. Using squared error loss function with prior distribution $\pi(\lambda) = e^{-\lambda}$ if $\lambda > 0$, find Bayes estimate of λ.

c) If the risk function is constant, show that the estimator is a minimax estimator.

(5 + 5 + 4 = 14)

8. a) Describe the method of Bayes estimation.

b) What do you mean by admissibility of estimators.

c) Let $X \sim b(n,\theta)$ and the loss function $L(\theta, d) = (\theta-d)^2/\theta(1-\theta)$. Using $\pi(\theta)=1$, $0 < \theta < 1$. Find a minimax estimate of θ.

(5 + 4 + 5 = 14)

UNIT V

9. a) Define (1-α)% confidence interval. Based on a sample of size 1 from the population $f(x; \theta) = \frac{2}{\theta^2}(\theta - x) \quad 0 < x < \theta$.

Find (1- α) level confidence interval for θ;

b) Let $X_1, X_2, \ldots X_m$ and $Y_1, Y_2, \ldots Y_n$ be i.i.d samples from $N(\mu_1, \sigma_1^2)$ and $N((\mu_2, \sigma_2^2)$ respectively. Find (1-α) confidence interval for σ_1^2/σ_2^2 when μ_1, μ_2 are unknown

c) Let X and Y be independent random variables with densities $\lambda e^{-\lambda x}$ and $\mu e^{-\mu y}$ $(x, y > 0)$ respectively. Find a (1- α) level confidence region for (λ, μ) of the term $\{(\lambda,\mu) : \lambda x + \mu y \le k\}$

(4 + 5 + 5 = 14)

Contd....3

10. a)

b)

c)

10. a) what do you mean by shortest length confidence interval. Let $f(x; \theta) = e^{-(x-\theta)}$, $x > \theta$. Find the shortest length confidence interval for θ at level $(1-\alpha)$ based on a sufficient statistic for θ.

b) Define unbiased confidence interval. Find an unbiased confidence interval for θ based on $X_{(n)}$ when $X \sim U(0, \theta)$.

c) Find a UMA family of confidence interval for θ when $X \sim U(0, \theta)$. Compare it with the unbiased confidence interval.

(5+ 5 + 4 =14)