M 22137

Reg. No. : ....

Name : ....

Third Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, November 2012
STATISTICS
Paper – 3.3 : Inference – II

Time: 3 Hours Max. Marks: 70

***Instructions :** Answer **any 5** questions without omitting **any** Unit.*
***All** questions carry **equal** marks.* **(5×14=70)**

UNIT – 1

I. a) Define (i) Critical region (ii) Type I error and (iii) Type II error. A sample of size 1 is taken from a population with Poisson distribution having mean θ . To test $H_0 : \theta = 1$ Vs $H_1 : \theta = 2$, consider the test $\phi(x) = 1$ if $x \geq 2$ and 0 if $x < 2$. Find the probabilities of type I and type II errors. If it is required to achieve a size equal to 0.05, how should you modify the test ?

b) State Neyman-Pearson lemma. Let x be a random variable with density $f(x) = \frac{1}{2\theta} e^{\frac{-|x|}{\theta}} \quad \infty < x < \infty; \theta > 0$. Find a most powerful test of size α based on a sample of size n, for testing $H_0 : \theta = \theta_0$ Vs $H_1 : \theta = \theta_1 (> \theta_0)$.

II. a) Show that one parameter exponential family satisfy monotone likelihood ratio property. How do you use this property to develop uniformly most powerful tests ?

b) Let $X_1, X_2 .... X_n$ be a sample of size n from U (0, θ), $\theta > 0$. Obtain a uniformly most powerful test for testing $H_0 : \theta = \theta_0$ Vs $H_1 : \theta \neq \theta_0$.

P.T.O.

UNIT – 2

III. a) Define (i) Unbiased test and (ii) Similar test. Let the power function of the test ϕ of $H_0 : \theta \in \Theta_0$ Vs $H_1 : \theta \in \Theta_1$ be continuous in θ. Show that if ϕ is UMP α-similar test then it is UMP unbiased test.

b) Distinguish between unbiased critical regions of type A and type B. Illustrate these with suitable examples.

IV. a) Explain locally most powerful tests. Let X be a random variable with Cauchy density having location parameter θ Obtain a locally most powerful test of $H_0 : \theta \leq 0$ Vs $H_1 : \theta > 0$.

b) Let $X \sim N\left(\mu_1, \sigma_1^2\right)$ and $Y \sim N\left(\mu_2, \sigma_2^2\right)$. Assume that X and Y are independent. Derive a UMP unbiased test of $H_0 : \sigma_1 - \sigma_2$ Vs $H_1 : \sigma_1 \neq \sigma_2$ based on two independent sets of samples.

UNIT – 3

V. a) Define likelihood ratio test. Obtain asymptotic distribution of the test statistic stating required conditions.

b) Let $X_1, X_2 \ldots\ldots X_k$ having jointly a multinomial distribution with probabilities $p_1, p_2, \ldots p_k$. Derive likelihood ratio test of $H_0 : p_i = p_{i0}$, i = 1, 2 .... k Vs $H_1 : p_i \neq p_{i0}$ for at least one i = 1, 2 .... k.

VI. a) Prove or disprove : Likelihood ratio tests are always UMP unbiased.

b) Let X be a random variable with exponential distribution $f(x\,;\theta,\beta) = \frac{1}{\theta} e^{-\frac{(x-\beta)}{\theta}}$ $x > \beta$. Derive likelihood ratio tests of

i) $H_0 : \beta = \beta_0$ Vs $H_1 : \beta \neq \beta_0$ (θ unknown)

ii) $H_0 : \theta = \theta_0$ Vs $H_1 : \theta \neq \theta_0$ (β unknown)

UNIT – 4

VII. a) Define SPRT. How do you determine stopping bounds of SPRT ? Describe its advantages over classical test.

b) Define O.C. function. Let X ~ P (θ). Derive SPRT of $H_0 : \theta = \theta_0$ Vs $H_1 : \theta = \theta_1$ ($> \theta_0$). Find O.C. function of the test.

VIII. a) Show that SPRT terminates with probability one.

b) Define ASN function. Let X ~ N (θ, 1). Derive SPRT of $H_0 : \theta = \theta_0$ Vs $H_1 : \theta = \theta_1$ ($> \theta_0$). Obtain ASN function of the test.

UNIT – 5

IX. a) Explain Kolmogrov-Smirnov test for goodness of fit. Show that the test statistic is distribution free.

b) Describe Wilcoxon signed rank test. Derive the distribution of the test statistic. Discuss its advantages over sign test.

X. a) Explain Mann-Whitney U-test. Compute mean and variance of the test statistic.

b) What do you mean by a non parametric confidence interval ? Construct a 100 (1 – α)% confidence interval for median of a population.