M 20149

Reg. No. : ....

Name : ....

Third Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, November 2011
STATISTICS
Paper – 3.3 : Inference – II

Time: 3 Hours | Max. Marks: 70

***Instructions** : Answer **any 5** questions without omitting **any** Unit.*
***All** questions carry **equal** marks.*

UNIT – 1

I. a) Define most powerful test. Consider the testing problem $H_0 : X \sim U(0, 1)$ Vs $H_1 : X \sim f(x)$

where $f(x) = 4x \; ; \; 0 < x < \frac{1}{2}$

$= 4 - 4x \; ; \; \frac{1}{2} \le x < 1.$

Obtain a most powerful test of size α. Find its power.

b) State and prove Neyman-Pearson lemma.

II. a) Define Monotone Likelihood Ratio (MLR) property. Prove or disprove : The family $N(\theta, \theta) \; ; \; \theta > 0$ satisfies MLR property.

b) Define UMP test. Let $X \sim N(\theta, \sigma^2)$. Prove that there does not exist UMP test of $H_0 : \sigma = \sigma_0$ Vs $H_1 : \sigma \neq \sigma_0$.

UNIT – 2

III. a) Define UMP unbiased test. Let X be a random variable with density $f(x; \theta) = e^{-(x-\theta)}, x > \theta$. Find a UMP unbiased test of size α of $H_0 : \theta = \theta_0$ Vs $H_1 : \theta \neq \theta_0$.

b) Define UMP α-similar test. Let power function of test ϕ be continuous in θ. If ϕ is an unbiased test of size α of $H_0 : \theta \in \theta_0$ Vs $H_1 : \theta \in \theta_1$, show that it is α-similar on the boundary.

P.T.O.

IV. a) Define locally most powerful test. Let X be a random variable with density

$f(x;\theta)=\frac{e^{x-\theta}}{(1+e^{x-\theta})^2}\ -\infty<x<\infty$. Obtain a locally most powerful test of $H_0:\theta\le 0$ Vs $H_1:\theta>0$.

b) Let X and Y be two random variables with exponential densities having means θ_1 and θ_2 respectively. Assume that X and Y are independent. Derive a UMP unbiased test of $H_0:\theta_1=\theta_2$ Vs $H_1:\theta_1\ne\theta_2$.

UNIT – 3

V. a) Describe likelihood ratio test. Explain the properties of the likelihood ratio test statistic.

b) Let $X_1, X_2, \ldots X_n$ be a sample from the probability mass function $P(x=k)=\frac{1}{N}\ k=1,2-N$ Obtain likelihood ratio test of $H_0:N=N_0$ Vs $H_1:N\ne N_0$.

VI. a) Derive asymptotic distribution of the likelihood ratio test statistic, stating regularity conditions.

b) Let $X_1, X_2, \ldots X_m$ and $Y_1, Y_2 \ldots Y_n$ be two independent random samples from two exponential distributions with density functions $f(x, a_1, b_1)=\frac{1}{b_1}e^{-\frac{1}{b_1}(x-a_1)}\ x>a_1$ and

$g(y, a_2, b_2)=\frac{1}{b_2}e^{-\frac{1}{b_2}(y-a_2)}\ y>a_2$.

Derive likelihood ratio tests of

i) $H_0:a_1=a_2$ Vs $H_1:a_1\ne a_2$

ii) $H_0:b_1=b_2$ Vs $H_1:b_1\ne b_2$.

UNIT – 4

VII. a) Explain SPRT. Let $X\sim b(1,\theta)$.

Derive SPRT for testing $H_0:\theta=\frac{1}{2}$ Vs $H_1:\theta=\frac{3}{4}$. Consider $\alpha=\beta=0.1$.

b) State and prove fundamental identity of SPRT.

VIII. a) Define (i) O.C. Function and (ii) ASN function. Let $X \sim N(\theta, 1)$. Derive SPRT for testing $H_0 : \theta = \theta_0$ Vs $H_1 : \theta = \theta_1 (> \theta_0)$. Compute O.C. function of the test.

b) Distinguish between ranking and selection problems. Explain a method for selecting a normal population with least variance among a class of k normal populations.

UNIT – 5

IX. a) Define Kolmogrov-Smirnov test of goodness of fit. Derive null distribution of the test statistic.

b) Explain Mann-Whitney U-test. Describe advantages of this test over median test. Find the asymptotic distribution of the test statistic U.

X. a) Distinguish between sign test and signed rank test. How do you construct $100(1-\alpha)\%$ confidence interval for median using the sign test statistic.

b) Explain Wald-Wolfowitz run test for comparing two distribution functions. Find the distribution of the test statistic.

(5×14=70)
