M 16673

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STATISTICS
Paper – 3.3 : Inference II

Time: 3 Hours Max. Marks: 70

Instructions : 1) Answer any 5 questions without omitting any Unit.
2) All questions carry equal marks.

UNIT – 1

I. a) Define power and size of the test. Let X~N $(\theta, 1)$. For testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 \,(> \theta_0)$, consider the test function

$$\phi(x_1,\dots x_n) = \begin{cases} 1 & \text{if } \bar{x} > \theta_0 + \frac{z_\alpha}{\sqrt{n}} \\ 0 & \text{if } \bar{x} \le \theta_0 + \frac{z_\alpha}{\sqrt{n}} \end{cases}$$

What is the size of the test ? Show that the power function of ϕ is nondecreasing in θ.

b) State Neyman-Pearson Lemma. Let $1-\beta$ be the power of a Neyman-Pearson size α test $(0<\alpha<1)$. Show that $1-\beta \ge \alpha$.

II. a) What do you mean by monotone likelihood ratio property ? Let $X \sim N(\theta, \theta^2)$. Examine whether $N(\theta, \theta^2)$ satisfy monotone likelihood ratio property or not ?

b) Find a UMP test for testing $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta$, based on a sample of size n, when $f(x;\theta) = \frac{1}{\theta}, 0 < x < \theta$.

P.T.O.

UNIT – 2

III. a) Define unbiased test. Let $X \sim P(\lambda)$. Find a UMP unbiased test of size α for testing $H_0 : \lambda \leq \lambda_0$ against $H_1 : \lambda > \lambda_0$.

b) What do you mean by locally most powerful test ? Let $X \sim N(\theta, 1)$. Consider the testing problem $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$. Show that locally most powerful test coincides with the UMP test.

IV. a) When do you say that a test has Neyman structure ? Prove that a necessary and sufficient condition for all similar tests to have Neyman structure with respect to a sufficient statistic T is that the family of distributions for T is boundedly complete.

b) Let $X_1, X_2 \ldots X_m$ and $Y_1, Y_2 \ldots Y_n$ be two independent random samples from two inverse Gaussian distributions $I(\mu, \sigma)$ and $I(v, \tau)$ respectively; where

$$I(\mu, \sigma) = \sqrt{\frac{\sigma}{2\pi x^3}}\, e^{\frac{-\sigma}{2x\mu^2}(x-\mu)^2}; x > 0 \text{ and}$$

$$I(v, \tau) = \sqrt{\frac{\tau}{2\pi x^3}}\, e^{-\frac{\tau}{2xv^2}(x-v)^2}; x > 0.$$ Show that there exist a UMP unbiased test for testing $H_0 : v = \mu$, against $H_1 : v \neq \mu$, if $\tau = \sigma$.

UNIT – 3

V. a) Explain likelihood ratio test. State its important properties.

b) Let $f(x, \theta) = e^{-(x-\theta)}$ $x > \theta$. Find a likelihood ratio test for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$ with size α.

VI. a) Find asymptotic distribution of likelihood ratio test statistic, stating required conditions.

b) Let $X \sim N\left(\mu_1, \sigma_1^2\right)$ and $Y \sim N\left(\mu_2, \sigma_2^2\right)$. X and Y are independent. Construct a likelihood ratio test for $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$.

UNIT – 4

VII. a) Define SPRT. Let X~P (λ). Find the SPRT of $H_0 : \lambda = 1$ against $H_1 : \lambda = 2$.

b) State and prove Wald's fundamental identity.

VIII. a) Define O.C. function and ASN function. Let $X \sim N(\theta, 1)$. Find OC function for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$, when $\alpha = \beta = 0.05$

b) Define a selection problem. Explain a method for selecting a 'best' mean normal population among a class of k normal population.

UNIT – 5

IX. a) Define Kolmogorov-Smirnov test for goodness of fit. Explain its advantages over chi-square goodness of fit.

b) Define Wald-Wolfowitz run test. Find the distribution of the test statistic.

X. a) Explain median test. What is the asymptotic distribution of the test statistic ?

b) Define Kendall's tau 'τ'. Explain its uses. Find a 100 $(1-\alpha)\%$ confidence interval for τ.
