M 15146

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree Examination, November 2008
Paper – 3.3 : STATISTICS
Inference – II

Time : 3 Hours Max. Marks : 70

Instructions : 1) Answer any 5 questions without omitting any Unit.
2) All questions carry equal marks.

UNIT – 1

I. a) Define :

i) Critical region

ii) Size of the test and

iii) Power of the test

b) Let X be a discrete random variable taken on the values 1, 2, 3, 4 with probabilities P_0 and P_1 under H_0 and H_1 respectively.

X	1	2	3	4
P_0	$\frac{2}{13}$	$\frac{4}{13}$	$\frac{3}{13}$	$\frac{4}{13}$
P_1	$\frac{4}{13}$	$\frac{3}{13}$	$\frac{2}{13}$	$\frac{4}{13}$

The critical region is given by X = 1 or X = 3. Find the size and power of the test.

II. a) State and prove Neyman-Pearson lemma.

b) Let $x_1, x_2 \ldots x_n$ be a random sample firm a p.m.f.

$$P(X = x) = \frac{1}{N};\ x = 1, 2 \ldots N,\ N \geq 1 \text{ is an integer.}$$

Find most powerful test for testing $H_0 : N = N_0$ against $H_1 : N = N_1\ (>N_0)$.

P.T.O.

UNIT – 2

III. a) Define monotone likelihood ratio property. Show that

$$f(x,\theta) = \frac{1}{2} e^{\frac{-|x-\theta|}{2}} \quad -\infty < x < \infty$$

satisfies monotone likelihood ratio property.

b) Define UMP test. Find a UMP test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$, where $f(x, \theta) = \frac{1}{\theta}; 0 < x < \theta$.

IV. a) Define UMP unbiased test. Find UMP unbiased test for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$, where μ is the mean of normal population with unknown variance σ^2.

b) What do you mean by α - similar test ? Show that if the power function is continuous then a UMP α - similar test is UMP unbiased test provided its size is α.

UNIT – 3

V. a) Define likelihood ratio test. State important properties of the test.

b) Let $X_1, X_2, \ldots X_n$ be a random sample with p.m.f.

$P(X = x) = e^{-(x-\theta)} \; x > \theta$

Find the likelihood ratio test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.

VI. a) Let $\lambda(x)$ be the likelihood ratio test statistic for testing $H_0 : \theta \in \Theta_0$ against $H_1 : \theta \in \Theta_1$. Under certain regularity conditions, show that $-2 \log \lambda(x)$ is asymptotically distributed as a X^2 - random variable with degrees of freedom equals to the difference between the number of independent parameters in Θ and the number in Θ_0 where Θ is the parameter space $\Theta = \Theta_0 \cup \Theta_1$.

b) Suppose $X \sim N(\mu_1, \sigma^2)$, $Y \sim N(\mu_2, \sigma^2)$ and X and Y are independent. Find a liklihood ratio test for testing $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$.

UNIT – 4

VII. a) Explain sequential probability ratio test (SPRT). Point out its advantages over classical tests.

b) Let $X \sim N(\theta, 1)$. Find the sequential probability ratio test of $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (> \theta_0)$.

VIII. a) State and prove Wald's fundamental identity.

b) Define OC function of sequential probability ratio test. Let $X \sim P(\theta)$. Find O.C. function for the test $H_0 : \theta = 1$ against $H_1 : \theta = 2$, when $\alpha = \beta = 0.05$.

UNIT – 5

IX. a) Explain Chi-square test for goodness of fit.

b) Define one sample Kolmogorov - Smirnov test statistic. Show that the statistic is distribution free.

X. a) Distinguish between sign test and signed rank test. Explain their usefulness.

b) Describe median test. Using this, find a 100 $(1 - \alpha)$% confidence interval for median of a population.
