Reg. No. : ....

Name : ....

M 13835

Third Semester M.Sc. Degree Examination, November 2007
STATISTICS
Paper – 3.3 : Inference – II

Time : 3 Hours Max. Marks : 70

Instructions : 1) Answer any 5 questions without omitting any Unit.
2) All questions carry equal marks.

UNIT – I

1. a) Define (i) Type I error and (ii) Type II error. Let X have the Bernoulli density with parameter θ. Based on 5 observations X_1, X_2, X_3, X_4 and X_5, the critical region for the test $H_0 : \theta = \frac{1}{2}$ against $H_1 : \theta = \frac{3}{4}$ is given by $\frac{\Sigma X_i}{5} > 0.6$. Obtain the size and power of the test.

   b) State Neyman-Pearson lemma for MP test. For MP, test, show that the power is larger than or equal to the size of the test.

2. a) Define monotone likelihood ratio (MLR) properly. Explain its role in testing problem.

   b) Let X have the density $f(x; \theta, \lambda) = \frac{1}{\theta} e^{-\frac{1}{\theta}(x-\lambda)}$ $x > \lambda$
   Determine a UMP test of size α for testing $\lambda = \lambda_0$ against $H_1 : \lambda \neq \lambda_0$; θ is unknown.

UNIT – II

3. a) Define α-similar test. Explain its role in testing problem.

   b) Explain UMP unbiased test. Let $X \sim N(\mu_1, \sigma^2)$ and $Y \sim N(\mu_2, \sigma^2)$. X and Y are independent. Obtain UMPU test of size α for $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$.

4. a) Explain the concept of Neyman structure. Give an example to a test which has Neyman structure.

   b) Distinguish between unbiased critical region of type A and type B. Examine the existence of type A, test for testing $H_0 : \sigma = \sigma_0$ against $H_1 : \sigma \neq \sigma_0$, when $X \sim N(\mu, \sigma^2)$.

P.T.O.

UNIT – III

5. a) Define likelihood ratio test. Prove or disprove : Likelihood ratio test is consistent.

b) Obtain likelihood ratio test for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$, when $X \sim N(\mu, \sigma^2), \sigma^2$ is unknown. Show that the test is unbiased.

6. a) Let $X_1, X_2, ..., X_n$ be a random sample from a density $f(x; \theta) = \frac{1}{\theta} e^{-x/\theta}$ $x > 0$, $\theta > 0$. Find the likelihood ratio test for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$.

b) Stating the assumptions derive asymptotic distribution of likelihood ratio test statistic.

UNIT – IV

7. a) Explain sequential probability ratio test. Obtain SPRT for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$ where θ is the mean of Poisson distribution.

b) State and prove Wald's identity.

8. a) Define OC function and ASN function of SPRT. Find OC function for testing $H_0 : \mu = 10$ against $H_1 : \mu = 15$ when $\alpha = \beta = 0.1$, where μ is the mean of normal population.

b) Show that SPRT terminates with probability one.

UNIT – V

9. a) Explain the Chi-square test for goodness of fit. Describe its limitations.

b) Explain sign test. Obtain confidence interval for median difference based on sign test.

10. a) Define Mann-Whitney U-test statistic for two sample problem. Obtain the distribution of test statistic.

b) Define one sample Kolmogorov-Smirnov test statistic. How do you use this for the construction of confidence interval for distribution function ?

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