Reg. No. : ....

M 12518

Name : ....

Third Semester M.Sc. Degree Examination, November 2006
STATISTICS
Paper – 3.3 : Inference II

Time: 3 Hours Max. Marks: 70

*Instructions: 1) Answer any 5 questions **without** omitting any Unit.*
*2) All questions carry **equal** marks.*

UNIT – I

I. a) Distinguish between :

i) Parametric and non-parametric tests.

ii) Randomized and non-randomized tests.

b) Let X has the density,

$$f(x;\theta) = \theta x^{\theta-1} I_{(0,1)}^{(x)}$$

To test $H_0 : \theta \le 1$ against $H_1 : \theta > 1$, a sample of size 2 is selected and the critical region is given by

$$C = \left\{ (x_1, x_2) \middle| \frac{3}{4} x_1 < x_2 \right\}.$$

Find the power and the size of the test.

II. a) State and prove generalized Neyman-Pearson lemma.

b) Let X ~ N (0, 1) under H_0 and X ~ C (0, 1) under H_1. Find the most powerful size α test of H_0 against H_1.

UNIT – II

III. a) Distinguish the following terms and give an example in each case :

i) Unbiased test

ii) UMP unbiased test

iii) LMP test.

b) Define α-similar test. Show that if the power function is continuous, then a UMP α-similar test is UMP unbiased provided its size is α.

P.T.O.

IV. a) When do you say that a test has Neyman structure ? Give an example to a test which has Neyman structure.

b) Let X be a random variable with d.f. $F_x(\theta)$, where $F_x(\theta)$ belongs to a family f_θ and let T be a sufficient statistic for f_θ. Prove that a necessary and sufficient condition for all similar tests to have Neyman structure with respect to T is that the family P^T of distributions of T is boundedly complete.

UNIT – III

V. a) Define likelihood ratio test. Show that for testing $H_0 : \theta \in \Theta_0$ against $H_1 : \theta \in \Theta_1$, the likelihood ratio test is a function of every sufficient statistic for θ.

b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a pmf.

$P(X = x) = \frac{1}{N}$, $x = 1, 2, \ldots, N$, $N \geq 1$ is an integer. Find the likelihood ratio test for testing $H_0 : N \leq N_0$ against $H_1 : N > N_0$. where N_0 is a specified integer.

VI. a) Let $\lambda(\underset{\sim}{x})$ be the likelihood ratio criterion for testing $H_0 : \theta \in \Theta_0$ against $H_1 : \theta \in \Theta_1$. Under certain regularity conditions show that $-2 \log \lambda(\underset{\sim}{x})$ is asymptotically distributed as a chi-square random variable with degrees of freedom equals to the difference between the number of independent parameters in Θ and the number in Θ_0, where Θ is the parameter space.

b) Describe how you test $H_0 : \sigma \geq \sigma_0$ against $H_1 : \sigma > \sigma_0$ in $N(\mu, \sigma^2)$ when μ is unknown.

UNIT – IV

VII. a) Define sequential probability ratio test. For a sequential probability ratio test with bounds A, B (A > B) and strength (α, β), show that

$$A \leq \frac{1-\beta}{\alpha} \text{ and } B \geq \frac{\beta}{1-\alpha}.$$

b) Let $X \sim Poi(\theta)$, $\theta > 0$. Find the sequential probability ratio test of $H_0 : \theta = 1$ against $H_1 : \theta = 2$.

VIII. a) Define OC function and ASN function of sequential probability ratio test. For testing $H_0 : \theta = 0.5$ against $H_1 : \theta = 0.9$, find OC function when $\alpha = \beta = 0.5$, where θ is the parameter in $b(1, \theta)$ and you are given a random sample of size n from $b(1, \theta)$.

b) Show that sequential probability ratio test terminates with probability one.

UNIT – V

IX. a) Define two sample Kolmogorov-Smirnov statistic. Describe its applications.

b) Show that the distribution of Spearman's rank correlation coefficient is symmetric about zero under independence.

X. a) Define Kendall's tan. Describe a test for its significance.

b) Explain Mann-Whitney U-test for the two sample problem.

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