Third Semester M.Sc. Degree Examination, November 2005
STATISTICS
Paper 3.3 : Inference II (2004 Admn.)

Time: 3 Hours Max. Marks: 70

Instructions : *Answer any 5 questions* ***without omitting*** *any Unit.* ***All*** *questions carry* ***equal*** *marks.*

UNIT – I

I. a) Describe the following terms :

i) randomized and non-randomized tests

ii) simple and composite hypothesis

iii) size and power of a test

b) Let X be a single observation from a population with pdf

$f(x\,;\,\theta) = \theta x^{\theta-1} I_{[0,\,1]},\ \theta > 0.$

Find a most powerful size α test of $H_0 : \theta = 2$ against $H_1 : \theta = 1$.

II. a) State and prove Neyman-Pearson fundamental lemma on testing of hypothesis.

b) When do you say that a family of pdf has MLR property ? Show that one-parameter exponential family has MLR property.

UNIT – II

III. a) Define UMP invariant test. Let $H_0 : X \sim N(\theta, 1)$ be tested against $H_1 = H\ X \sim C(1, \theta)$. A sample of size 2 is available. Find a UMP invariant test of H_0 against H_1.

b) Describe a situation where no UMP test exist . Illustrate it.

IV. a) Define the following terms and give an example in each case

i) UMP α- similar test

ii) Unbiased critical region of type A.

b) Show that if the power functions of a test is continuous then a UMP α- similar test is UMP unbiased provided its size is α.

UNIT – III

V. a) Define likelihood ratio test. Let (X_1, X_2 .... X_n) be random sample from a population with probability density function

$$f(x, \theta) = \begin{cases} e^{-(x-\theta)} & , x \geq \theta \\ 0 & , x < \theta \end{cases}$$

$-\infty < \theta < \infty$. Construct likelihood ratio test for testing $H_0 = \theta \leq \theta_0$ against $H_1 = \theta > \theta_0$, where θ_0 is a specified value.

b) What are the properties of likelihood ratio tests ?

VI. a) Let (X_1, X_2, ...X_m) and (Y_1, Y_2, ...Y_n) be independent random samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively. Construct likelihood ratio test for testing $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 = \sigma_1^2 \neq \sigma_2^2$.

b) Let (X_{12} .....X_{k-1}) be a multinomial random vector with parameters n, P_1, P_2, ...P_{k-1}. Show that the random variable

$$U_k = \sum_{i=1}^{k} \frac{(X_i - nP_i)^2}{nP_i}$$

is asymptotically distributed as a chi-square random variable with k–1 degrees of freedom.

UNIT – IV

VII. a) Define SPRT. Construct SPRT for testing $H_0 : \theta = 0.1$ against $H_1 : \theta = 0.5$ where θ is the parameter of a Bernoulli random variable.

b) Show that with probability one SPRT terminates.

VIII. a) State and prove Walds identity.

b) Show that the power function $\beta(\theta)$ of SPRT is

$$\beta(\theta) = \frac{1 - B^{t^*}}{A^{t^*} - B^{t^*}}$$

Where t^* is the solution of $E(e^{tz}) = 1$.

UNIT – V

IX. a) Define Kolmogorov-Smirnov statistics D_n, D_n^+, D_n^-. Show that they are completely distribution free for any continuous distribution F.

b) Explain Wilcoxon signed-rank test. What are the properties of this test ?

X. a) Define Mann-Whitney U-statistic. What is it used for ? Derive the mean and variance of U.

b) Describe applications of Kolmogorov-Smirnov one sample statistic.