M 24480

Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com./M.Sc. Computer Science Degree (Regular/Supplementary/Improvement) Examination, November 2013

STATISTICS

Paper – 1.4 : Distribution Theory

Time : 3 Hours Max. Marks : 60

Instructions : *Answer* ***any five*** *questions without omitting* ***any*** *unit.*
All *questions carry* ***equal*** *marks.*

UNIT – 1 **(1×12=12)**

1. a) Define geometric distribution. Obtain its probability generating function. Let X and Y be independent geometric random variables. Show that X – Y and min(X, Y) are independent.

   b) Define power series family of distributions. Derive recurrence relationships among its central moments.

2. a) Derive cumulants of power series family of distributions. Show that the cumulants satisfy the relationship $k_{r+1} = \theta \frac{dk_r}{d\theta}$ r = 1, 2...

   b) State and prove lack of memory property for geometric distribution.

UNIT – 2 **(1×12=12)**

3. a) Let X follows gamma distribution with parameters α and β and let Y ~ U (0, X). Find (i) the density of Y (ii) The conditional density of X given Y = y and (iii) P (X + Y < 2).

   b) Define Weibull distribution. Obtain its mean and variance. Let X_i (i = 1, 2 ...n) be i.i.d. random variables. Show that min (x_1, x_2, ... x_n) has Weibull distribution if the original distribution is Weibull.

P.T.O.

4. a) Define Laplace distribution. Obtain its moment generating function. Hence or otherwise find E(X) and V(X).

   b) Distinguish between type I and type II censoring schemes. Write down the likelihood function in each case, when the original distribution is exponential with mean θ.

UNIT – 3 **(1×12=12)**

5. a) Define Pearson family of distributions. Show that normal and gamma distributions belong to this family.

   b) Define beta distribution. Show that it is a particular case of type I Pearson distribution.

6. a) Describe the procedure for classifying the Pearson family of distributions into various types.

   b) Define Burr family of distributions. What are various types of distributions belonging to this family ?

UNIT – 4 **(1×12=12)**

7. a) State and prove Fisher-Cochran theorem.

   b) Derive non-central 't' distribution.

8. a) If X_i (i = 1, 2, ... n) are n independent normal variates with zero means and unit variances, show that $\sum_{i=1}^{n} X_i$ and $\sum_{i=1}^{n} (X_i - \overline{X})^2$ are independent. Hence obtain the distribution of $U = \dfrac{\sum^{n} X_i}{\sqrt{\Sigma(X_i - \overline{X})^2}}$.

   b) If X and Y are independent Chi-square variates with n_1 and n_2 degrees of freedom, then show that U = X + Y and $V = \dfrac{n_2 X}{n_1 Y}$ are independent. Find the distribution of V.

UNIT – 5   (1×12=12)

9. a) Let $X_{(1)}, X_{(2)}, \ldots X_{(n)}$ be the set of order statistics of $X_1, X_2, \ldots X_n$ with common density as exponential with mean θ. Show that $X_{(r)}$ and $X_{(s)} - X_{(r)}$ are independent for any s > r. Find the distribution of $X_{(r+1)} - X_{(r)}$.

b) Let $X_1, X_2, \ldots X_n$ be i.i.d. non-negative random variables with distribution function F(x). If $L^2|X| < \infty$. Show that $E|X_{(r)}| < \infty$. Also show that

$$E\,M_n - E\,M_{n-1} + \int_0^\infty F^{n-1}(x)(1-F(x))dx \text{ where } M_n = \max(X_1, \ldots X_n);\ n = 2, 3 \ldots$$

10.a) Derive the distribution of sample median when samples are drawn from an exponential distribution with mean θ.

b) Let $X_{(1)}, X_{(2)} \ldots X_{(n)}$ be the order statistics based on a random sample from uniform distribution with density f(x) = 1, 0 < x < 1. Show that $Y_1 = \frac{X_{(1)}}{X_{(2)}}, Y_2 = \frac{X_{(2)}}{X_{(z)}}, Y_{(n-1)} = \frac{X_{(n-1)}}{X_{(n)}}$ and $Y_n = X_{(n)}$ are independent.
