M 22390

Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com. Degree (Reg./Supple./Imp.) Examination, November 2012
STATISTICS
(Paper – 1.4) : Distribution Theory

Time : 3 Hours

Max. Marks : 60

Instruction : *Answer* ***any five*** *questions without omitting any Unit.* ***All*** *questions carry* ***equal*** *marks.*

UNIT – I

(1×12=12)

I. a) If X is a Poisson variate with parameter λ show that $P[X \le k] = \sum_{x=0}^{k} \frac{e^{-\lambda}\lambda^{x}}{x!} = 1 - I_{\lambda}(k+1)$ where $I_{\lambda}(k+1) = \frac{1}{\sqrt{k+1}} \int_{0}^{\lambda} e^{-\xi} \xi^{k} d\xi$ is an incomplete gamma function.

b) Define a power series distribution. Obtain the logarithmic series distribution from the power series distribution and hence obtain its expectation.

II. a) Define the negative binomial distribution. Show that it can be obtained by compounding the Poisson distribution with the Gamma distribution.

b) Distinguish between a binomial and hypergeometric variate. Stating the conditions clearly under which a binomial variate tends to a Poisson variate, prove the same.

UNIT – II

(1×12=12)

III. a) Let X have density $f(x) = \begin{cases} \frac{\beta\alpha^{\beta}}{x^{\beta+1}} & x \ge \alpha,\ \alpha > 0,\ \beta > 0 \\ 0 & \text{otherwise} \end{cases}$

Show that Ex^{n} exists if and only if $n < \beta$. If $\beta > 2$, find the mean and variance of the distribution.

b) Define a truncated distribution. Give the zero truncated binomial distribution. Compute its mean and variance.

P.T.O.

IV. a) If $X_1, X_2, ...X_n$ are independent normal random variables and $\sum_1^n a_i b_i \ V(X_i) = 0$ then show that $L_1 = \sum_{i=1}^n a_i X_i$ and $L_2 = \sum_{i=1}^n b_i X_i$ are independent, here a_i, b_i are fixed non zero real numbers.

b) Define a compound distribution. Show that the Pareto type II distribution is obtained by compounding the exponential distribution with gamma distribution.

UNIT – III (1×12=12)

V. a) Prove or disprove Cauchy and Logistic distributions are members of Pearson system of distributions.

b) Define the Bun family of distributions. Check whether the Beta distribution and logistic distribution are members of the Bun family.

VI. a) The probability density function f(x) of a random variable X defined over (–1, 1) satisfies the differential equation $\frac{df(x)}{x} = x(1-x^2)^{-1} f(x)$. Obtain the density function in an explicit form and identify it with one of Pearsonian types.

b) Let X and Y be independent $G(\alpha_1, \beta)$ and $C(\alpha_2, \beta)$ respectively. Then show that the conditional distribution (X|X+Y) is a Beta (α_1, α_2).

UNIT – IV (1×12=12)

VII. a) If X is a p-variate normal vector with expectation zero and dispersion matrix I_r derive the condition for the quadratic form X^1AX of rank r to be distributed as χ^2.

b) Derive the non-central χ^2 distribution. Explicitly state its non-centrality parameter.

VIII. If $(X_1, X_2, ...X_n)$ is a random sample from the distribution N (0, 1) and if $\sum_1^n \chi_i^2 = \sum_1^k Q_i$ where Q_i is a non-negative quadratic form in $X_1, X_2, ...X_n$ whose matrix has rank n_i, then prove that a necessary and sufficient condition for Qi to be independently distributed as $\chi^2(n_i)$ i = 1, ... k is that $\sum_1^k ni = n$.

UNIT – V (1×12=12)

IX. a) Derive the distribution function of range of a random sample of size n from a continuous distribution. From that deduce the median of sample range of a sample drawn from a uniform distribution in the interval (0, 1).

b) If $X_{(1)}, X_{(2)}, \ldots X_{(n)}$ are the set of order statistics of independent random variables $X_1, X_2 \ldots X_n$ with common pdf $f(x) = \begin{cases} \beta e^{-x\beta} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$ and $Z_1 = n X_{(1)}, Z_2 = (n-1)(X_{(2)} - X_{(1)}), Z_3 = (n-2)(X_{(3)} - X_{(2)}) \ldots Z_n = (X_{(n)} - X_{(n-1)})$ then show that $Z_1, Z_2, \ldots Z_n$ and $X_1, X_2, \ldots X_n$ are identically distributed.

X. a) Let $X_1, X_2, \ldots X_n$ be independent and identically distributed with

$$F(y) = \begin{cases} y^{\alpha} & 0 < y < 1, \alpha > 0 \\ 0 & \text{otherwise} \end{cases}$$

Show that $X_{(i)}/X_{(n)}, i = 1, 2, \ldots n-1$ and $X_{(n)}$ are independent.

b) Let X and Y be independent and identically distributed rvs. Then prove that X – Y is a symmetric random variable.
