(16)

M 18462

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, November 2010
STATISTICS
Paper – 1.4 : Distribution Theory

Time: 3 Hours

Max. Marks: 60

Instructions: *1) Answer **any five** questions **without** omitting any Unit.*
*2) **All** questions carry **equal** marks.*

UNIT – 1

1. a) If $X_1, X_2, \dots X_n$ are i.i.d. random variables with $P(X_1 = j) = \frac{1}{m}$, $j = 1, 2 \dots m$, find the probability generating function of $S_n = X_1 + \dots + X_n$ and hence the p.m.f. of S_n.

   b) X and Y are independent random variables with X following Poisson (λ) and Y following Poisson (μ). Obtain the conditional distribution of X given $X + Y = t$ and compute the mean of the conditional distribution.

2. a) Explain the logarithmic series distribution and discuss its distributional characteristics. Also deduce their particular cases.

   b) Show that a discrete random variable with support {0, 1, 2, ...} follow the geometric distribution if and only if the relationship

   $P\{X > m + n / X > m\} = P\{X \geq n\}$

   holds for positive integers m and n.

P.T.O.

UNIT – 2

3. a) If X follow the Pareto distribution with density

$f(y) = \theta k^{\theta} y^{-(\theta+1)}; y > k, k > 0, \theta > 0$, show that

i) $X = \log\left(\frac{Y}{k}\right)$ follow the exponential distribution

ii) $X = -\log\left[\left(\frac{Y}{k}\right)^{\theta} - 1\right]$ follow a logistic distribution.

b) Define the Laplace distribution. Obtain its characteristic function. Comment on the skewness and kurtosis of the distribution.

4. a) If the conditional distribution of X given θ is Poisson (θ) and θ has a Gamma distribution, obtain the distribution of X.

b) Define the Weibull distribution. Explain how it can be derived by transformation from an exponential random variable. Derive the characteristic function of a Weibull distribution. Hence or otherwise determine the first four raw moments of the distribution.

UNIT – 3

5. a) Write down the differential equation satisfied by the members of the Pearson family of distributions. Describe how will you classify the members of the family into various types, depending on the roots of a quadratic equation. Under what conditions can this reduce to the Gamma distribution ?

b) Define the Burr family of distributions and discuss on the important distributions belonging to this family.

6. a) The probability density function f(x) of a random variable X defined over (–1, 1) satisfies the differential equation

$$\frac{d\,f(x)}{dx} = \left[x\left(1 - x^2\right)\right]^{-1} f(x)$$

Obtain the density function in explicit form.

b) X is a random variable following the standard beta distribution, B (p, q). Show that the transformation $T = X/1 - X$ transforms the distribution to Pearson type VI distribution.

UNIT – 4

7. a) State and prove Cochran's theorem.

b) In sampling from a normal population, $N(\mu, \sigma^2)$, show that $(n-1)S^2/\sigma^2$ is distributed as a Chisquare with (n – 1) degrees of freedom where $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$.

8. a) Define the F statistic and derive its distribution. If F is F_{n_1, n_2}, what will be the distribution of $\frac{1}{F}$?

b) Define the non central t and obtain its distribution.

UNIT – 5

9. a) If $X_{r:n}$ is the r^{th} order statistic based on a random sample of size n (≥ 2) and if $\mu_{r:n}^{(k)} = E\left(X_{r:n}^{k}\right)$, then prove that

$$r\,\mu_{r+1:n}^{(k)} + (n-r)\mu_{r:n}^{(k)} = n\,\mu_{r:n-1}^{(k)}, 1 \leq r \leq n-1.$$

b) Let $X_{1:n}, X_{2:n} \dots X_{n:n}$ be the order statistics based on a random sample of size n from a Bernoulli distribution with p as the probability of success. Obtain the p.d.f. of $X_{r:n}$, $E(X_{r:n})$ and $V(X_{r:n})$.

10. a) Define 'standard error'. Obtain the standard error of the F statistic with (n_1, n_2) d.f. Comment on the standard error of n_1F as $n_2 \to \infty$.

b) Let $X_1, X_2, \dots X_n$ be i.i.d. random variables with distribution function

$F(x) = 1 - e^{-bx}, x > 0$.

If $X_{(r)}$ is the r^{th} order statistic, obtain the distribution of

$d_r = X_{(r+1)} - X_{(r)}, r \geq 0, X_{(0)} = 0$.
