M 16934

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, November 2009
STATISTICS
Paper – 1.4 : Distribution Theory

Time: 3 Hours Max. Marks: 60

***Instructions:** 1) Answer **any five** questions without omitting **any** Unit.*
*2) **All** questions carry **equal** marks.*

UNIT – 1

I. a) Let X be a random variable with p.m.f.

$P(X = -2) = \frac{1}{5},\ P(X = -1) = \frac{1}{6},\ P(X = 0) = \frac{1}{5},\ P(X = 1) = \frac{1}{15}$ and

$P(X = 2) = \frac{11}{30}$. Obtain the p.m.f. of $Y = X^2$.

b) Derive the negative bionomial distribution and obtain the generating function. If $X_1, X_2, \ldots, X_k$ are independent NB $(r_i; p)$ r.v.'s, i = 1, 2, ... k respectively, show that $S_k = \sum_{i=1}^{k} X_i$ is distributed as $NB(r_1 + \ldots + r_k; p)$.

II. a) Define cumulant generating function and obtain it for the Poisson (λ) distribution. Deduce the mean and variance.

b) Consider the trinomial distribution with p.m.f.

$$P\{X = x, Y = y\} = \frac{n!}{x!\,y!\,(n-x-y)!} p_1^x p_2^y p_3^{n-x-y}$$

Where x and y are non-negative integers such that $x + y \le n$ and $p_1, p_2, p_3 > 0$ with $p_1 + p_2 + p_3 = 1$. Obtain the marginal distributions and conditional distributions. Also evaluate the conditional expectation E(Y/x).

P.T.O.

UNIT – 2

III. a) If X follow the Pareto I distribution, show that the relationship $G(st) = G(s)\,G(t)$ holds for all $s \geq 1$ and given t, where $G(x) = P(X > x)$.

b) If $X_1, X_2, \ldots X_n$ are independent and identically distributed random variables following the Weibull distribution, show that $Y = \min(X_1, X_2, \ldots X_n)$ also follow the Weibull, except possibly for a change in the parameter.

IV. a) Distinguish between censoring and truncation. If X is a non-negative random variable with distribution function F(x), write down the distribution function of $Y = X - t/X > t;\ t > 0$.

b) If X and Y are random variables following the uniform (0, 1) distribution, find the distribution of (X – Y).

UNIT – 3

V. a) Define the Pearson family of distributions. Identify two distributions which (1) belongs to the family and (2) does not belong to the family. How will you classify the different members of the family ?

b) Explain the mathematical forms of the Burr family of distributions and state its properties.

VI. a) Describe the basic structures of the following families of distributions with examples :

1) GLD family

2) Beta family.

b) What are orthogonal polynomials ? Describe the utility of orthogonal polynomials in statistical theory.

UNIT – 4

VII. a) Obtain a necessary and sufficient condition for the independence of two quadratic forms $X'AX$ and $X'BX$.

b) Define the Students t statistic and derive its distribution. Identify the distribution of T^2, where T is the Students t statistic.

VIII. a) Derive the non-central Chi-square distribution and write down the expression for the moment generating function.

b) In sampling from a normal population, show that the sample mean, $\overline{X}$ and the sample variance S^2 are independently distributed.

UNIT – 5

IX. a) If $X_1, X_2, \ldots, X_n$ are i.i.d. random variables and $X_{(1)} = \min(X_1, X_2, \ldots, X_n)$, show that $nX_{(1)}$ is distributed as X_1 if and only if X_1 is distributed as exponential.

b) Obtain the asymptotic distribution of the sample median in sampling from an exponential distribution.

X. a) If $X_1, X_2, \ldots, X_n$ is a random sample of size n from an absolutely continuous distribution F(x) with p.d.f. f(x), obtain the expression for the distribution of the sample range.

b) Describe in detail any one method for the generation of order statistics from a given population.

M-5-1

$M_{X_1 \cdots X_k}(t_1 \cdots t_k) = E[e^{t_1x_1 + t_2x_2 + \cdots + t_kx_k}]$

$= \sum_{x=0}^{n} \cdots \sum_{k=0}^{n} e^{t_1x_1 + t_2x_2 + \cdots + t_kx_k} \cdot \frac{n!}{x_1! x_2! \cdots x_k!} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k}$

$= (p_1e^{t_1} + p_2e^{t_2} + \cdots + p_ke^{t_k})^n$

mean & variance of multinomial dist

$M_X(t) = [p_1e^{t_1} + \cdots + p_ke^{t_k}]^n$ $E(X_i) = \frac{\partial}{\partial t} M_X(t)/_{t=0}$

$E(X_i) = \frac{\partial}{\partial t} M_X(t)/_{t=0}$ $= n[p_1e^{t_1} + \cdots + p_ke^{t_k}]^{n-1} p_i e^{t_i}$

$= np_ie^{t_i}[p_1e^{t_1} + \cdots + p_ke^{t_k}]^{n-1}$

$E(X_i^2) = \frac{\partial^2}{\partial t^2} M_X(t)/_{t=0}$

$E(X_iX_j) = \frac{\partial}{\partial t_i \partial t_j} M_X(t)/_{t=0}$ $\frac{\partial}{\partial t} M_X(t)/_{t=0} = np_i e^0 [p_1 + \cdots + p_k]^{n-1}$

$= np_i$