M 15470

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, December 2008
STATISTICS
Paper – 1.4 : Distribution Theory

Time : 3 Hours   Max. Marks : 60

Instructions : *1) Answer any five questions without omitting any Unit.*

2) All questions carry equal marks.

UNIT – 1

I. a) Define probability generating function for an integer valued random variable. If X is a non-negative inter valued random variable with p.g.f., P(s), show that $\sum_{n=0}^{\infty} s^n P(X \leq m) = \frac{P(s)}{1-s}$.

b) Define the power series distribution and obtain the expression for the moment generating function. Deduce the mean and variance.

II. a) If the conditional distribution of X given θ is Poisson (θ) and θ has Gamma distribution. Show that the distribution of X is negative binomial.

b) X and Y are independent random variables following negative binomial distributions, NB (r_1 ; p) and NB (r_2, p) respectively. Obtain the conditional p.m.f. of X given X + Y = t. Deduce the corresponding result for the geometric distribution.

UNIT – 2

III. a) Derive the lognormal distribution from the normal distribution. For the lognormal distribution, show that Mean > Median > Mode.

b) Define the two parameter Weibull distribution and obtain its mean and variance. Write down the transformation that produces an exponential random variable. If X follow the Weibull distribution, derive the distribution of $y = -\log X$.

IV. a) Let (x, y) be jointly distributed with density $f(x, y) = \begin{cases} e^{-y}, & 0 < x < y < \infty \\ 0, & \text{Otherwise} \end{cases}$

Obtain :

1) The marginal densities and

2) The conditional densities. Also find $E(y \mid x)$.

b) Distinguish between compound and mixture distributions. Give a brief outline of the use of mixture distributions in modelling statistical data.

UNIT – 3

V. a) Explain the basic structure of the Burr family of distributions. Identify two distributions which are members of the family. Also state any two properties of the Burr family of distributions.

b) Prove or disprove : Cauchy and logistic distributions are members of the Pearson system of distributions.

VI. a) Explain the need for studying family of distributions from the point of view of modelling statistical data. Also define orthogonal polynomials and discuss its use.

b) Describe the basic structures of :

1) GLD family and

2) Beta family with examples.

UNIT – 4

VII. a) Define an F statistic. Obtain its probability distribution and hence derive an expression for the r^{th} central moment.

b) If X follow N_p (0, I), obtain the distribution of the quadratic form $X'AX$.

VIII. a) Define the non-central t statistic and the non-central F statistic. Show that the square of a non-central t statistic is a non-central F statistic. Also mention an application, each of the above distributions.

b) State and prove the Fisher-Cochran theorem.

UNIT – 5

IX. a) Define order statistics. Obtain the joint distribution of any two order statistics $(X_{(r)}, X_{(s)})$

b) If $X_{(1)}, \ldots\ldots X_{(n)}$ are order statistics based on a random sample from

$f(x) = \theta e^{-\theta x}$, $0 < x < \infty$, find the distribution of

$Z_i = (n - i + 1)(X_{(i)} - X_{(i-1)}), i = 1, 2 \ldots n, X_{(0)} = 0.$

X. a) Obtain the asymptotic distribution of the sample range in sampling from an exponential population.

b) Define standard error. Obtain the standard error of a students t statistic with n (≥ 3) degrees of freedom.