Reg. No. : ....

Name : ....

M 13936

First Semester M.Sc. Degree Examination, November 2007
STATISTICS
Paper – 1.4 : Distribution Theory

Time: 3 Hours

Max. Marks: 60

*Instructions :1) Answer **any five** questions without omitting **any** Unit.*
*2) **All** questions carry **equal** marks.*

UNIT – I

I. a) Define probability generating function (pgf) of a random variable (rv). Let X be an rv with pmf. $P(X = j) = P_j$; $j = 0, 1, 2, \ldots\ldots$ set $P(X > j) = q_j$, $j = 0, 1, 2, \ldots..$ Write $Q(s) = \sum_{j=0}^{\infty} q_j s^j$. Then show that $Q(s) = \frac{1-P(s)}{1-s}$ for $|s| < 1$, where P(s) is the pgf of X. Find the mean and the variance of X.

b) Define hypergeometric distribution. Derive the moment generating function (mgf) of it and hence obtain its mean and variance.

II. a) Define modified power series family of distribution. Derive the reasonable relationship connecting the cumulants of it and hence obtain the first four moments of binomial distribution.

b) If X_i $(i = 1, 2, \ldots n)$ be independently distributed with Poison distribution $P(\lambda_i)$ and if $T_0 = \sum X_j$, $\lambda = \sum \lambda_i$ show that T_0 has the Poison distribution $P(\lambda)$ and the conditional distribution of $x_1, x_2, \ldots\ldots x_{n-1}$ given $T = t_0$ is a multinomial.

UNIT – II

III. a) Show that for the Pareto distribution,

i) $P(X > xu \mid X > u) = P(X > u)$ for all $x, u > 1$

ii) $E\left(\frac{X}{x} \middle| X > x\right) = E(X)$ for all $x > 1$.

b) Define two parameter logistic distribution. Obtain the mgf of this distribution and hence deduce mean and variance. Also show that if X has a pareto distribution with parameters θ and α, then $Y = ln\left(\frac{X}{\theta}\right)$ is logistic.

IV. a) Define two parameter Laplace distribution. Obtain the mgf of the distribution and hence find its mean and variance.

b) If X and Y are i.i.d. normal random variables with zero mean and unit variance, obtain the distribution of $Z = \frac{XY}{\sqrt{X^2 + Y^2}}$.

UNIT – III

V. a) Let $X \sim G(\alpha_1, \beta)$ and $Y \sim G(\alpha_2, \beta)$ be independent random variables. Then X + Y and X/X + Y are independent.

b) Stating clearly the assumptions, derive the differential equation satisfied by the pearsonian system of frequency curves. Prove or disprove "Cauchy and Logistic distributions are members of Pearsons system of distributions".

VI. a) Let X_1, X_2 be independent random variables fallowing Beta distribution with parameters (α_1, β_1) and (α_2, β_2) respectively. Then show that X_1X_2 is also Beta distribution.

b) Explain the principle involved in generating the Burr family of distributions. Prove or disprove 'Logistic distribution is a member of the Burr family of distributions'.

UNIT – IV

VII. a) State and prove Fisher-Cochran theorem.

b) Let $X_1, X_2, \ldots X_n$ be i.i.d. random variables following the normal distribution. Show that the sample mean and sample variance are independently distributed. Hence or otherwise obtain their distributions.

VIII. a) Derive the distribution of non-central chi-square.

b) Define an F-statistic. Obtain its probability distribution. If X is distributed as $F(r_1, r_2)$, show that $Y = {}^1\!/_X$ is distributed as $F(r_2, r_1)$.

UNIT – V

IX. a) Derive the sample distribution of the range in a random sample of size n from $f(x) = \theta e^{-\theta x}$; $x > 0$, $\theta > 0$.

b) Obtain the asymptotic distribution of the sample median from an exponential population.

X. a) Let $X_{(1)}, X_{(2)}, \ldots X_{(n)}$ be the order statistics in a random sample of size n from the exponential distribution. Show that $(Z_1, Z_2, \ldots Z_n)$ and $(X_1, X_2, \ldots X_n)$ are identically distributed where $Z_1 = nX_{(1)}$, $Z_{(i)} = (n - i + 1)(X_{(i)} - X_{(i-1)})$.

b) Let $X_1, X_2, \ldots X_n$ be a random sample of size n from a distribution $F(x; \theta)$. Let $m_r = \frac{1}{n}\sum x_i - \bar{x})^r$ be the v^{th} sample moment about sample mean. Derive the large sample variance of m_r.

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