15/12/02 (79)

Reg. No. : ....

Name : ....

M 12703

First Semester M.Sc. Degree Examination, November 2006
STATISTICS
Paper – 1.4 : Distribution Theory

Time: 3 Hours Max. Marks: 60

Instructions: 1) *Answer **any five** questions without **omitting** any Unit.*
2) ***All** questions carry **equal** marks.*

UNIT – I

I. a) For a negative binomial distribution, derive the cumulant generating function and hence obtain the recurrence relation among its cumulants.

b) The number of female birds of a rare species in a region follows a Poisson distribution with mean m. The number of eggs laid by each female bird also follows a Poisson distribution with parameter λ. Find the probability distribution of X, the number of eggs of the rare birds in the region. Find also the mean and variance of X.

II. a) Let X be a non negative integer valued random variable then show that the condition

$$P(X > m + n \mid X > m) = P(X > n)$$

Geometric distn (Memory Prop)

for any two positive integers m and n is satisfied if and only if X has a geometric distribution.

b) Derive multinomial distribution. Obtain the m.g.f. of multinomial distribution and hence deduce the mean and variance of it.

UNIT – II

III. a) For the logistic distribution with c.d.f. $F(x) = (1 + e^{-x})^{-1}$ and p.d.f. f(x), show that $x = \log\left(\frac{F(x)}{1-F(x)}\right)$ and $f(x) = F(x)\,[1 - F(x)]$.

b) Define Weibull distribution. Let X_i, i = 1, 2, ..., n are i.i.d. random variable. Show that Min $(X_1, X_2, ..., X_n)$ has a Weibull distribution if and only if each X_i has a Weibull distribution.

P.T.O.

IV. a) Explain the concepts of compound and mixture distribution. Show that Laplace distribution as a mixture of two pdfs. Also obtain the mean and variance.

b) Derive Lognormal distribution. Obtain the mgf of Lognormal distribution. If X has a Lognormal distribution, find the distribution of X^b.

UNIT – III

V. a) Derive the characteristic function of $f(x) = \frac{m^P}{\Gamma(P)} x^{P-1} e^{-mx}, x > 0 ; m, P > 0$ and also find its r^{th} cumulant.

b) Obtain the differential equation satisfied by the Pearsonian system of frequency distributions, stating the assumptions underlying. State how it leads to different distributions. Show further that all Pearsons distributions are determined by the first four moments.

VI. a) Let X and Y be independent $G(\alpha_1, \beta)$ and $G(\alpha_2, \beta)$ respectively, then show that $X/X + Y$ is a $B(\alpha_1, \beta_2)$. Obtain its mean and variance.

b) Define Burr family of distributions and mention the important distributions belonging to this family.

UNIT – IV

VII. a) Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution. Show that $\sum X_i$ and $\sum (X_i - \overline{X})^2$ are independent.

b) Define central t-statistics. Derive the moments of its distribution. Mention some applications of this statistic.

VIII. a) Derive the distribution of non-central F statistic.

b) Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with common distribution U(0, 1), then show that $-2\sum_{i=1}^{n} \log X_i$ is $X^2(n)$. Derive the m.g.f. of the distribution and hence find its mean and variance.