First Semester M.Sc. Degree Examination, November 2005
STATISTICS
Paper 1.4 : Distribution Theory

Time: 3 Hours Max. Marks: 60

Instructions : *1) Answer any five questions without omitting any Unit.*
*2) All questions carry **equal** marks.*

UNIT – I

I. a) Define p.g.f. of a non-negative integer valued random variable. Let X_i, i = 1, 2, ... N be i.i.d r.v.'s with p.g.f. P (S), where N is also a r.v. independent of X_i's. Obtain the p.g.f. of $\sum_{i=1}^{N} X_i$.

b) Define Generalised Power Series Distribution (GPSD). Hence deduce Logarithmic series distribution and obtain its mean and variance.

II. a) Derive the moment generating function of hypergeometric distribution and state its properties.

b) If X_i, i = 1, 2, ... n be independently distributed with Poisson distribution P (λ_i) and if $T = \sum_{i=1}^{N} X_i$, $\lambda = \sum_{i=1}^{N} \lambda_i$, show that T has the Poisson distribution P (λ) and the conditional distribution of $X_1, X_2, ... X_{n-1}$ given T = t is a multinomial.

UNIT – II

III. a) For the logistic distribution with c.d.f. $F(x) = (1 + e^{-x})^{-1}$ and p.d.f f(x), show that $x = \log\left[\frac{F(x)}{1-F(x)}\right]$ and $f(x) = F(x)\ [1 - F(x)]$.

b) Let X and Y be independent r.v.'s with common p.d.f. $f(x) = \beta^{-\alpha}\ \alpha x^{\alpha-1}$ if $0 < x < \beta$ and = 0 otherwise ; $\alpha \geq 1$. Let U = min (X, Y) and V = max (X, Y). Find the joint p.d.f. of U and V and the p.d.f. of U + V. Show that U/V and V are independent.

IV. a) Derive compound exponential distribution and hence obtain the moments.

b) Define Weibull distribution. Explain how it can be derived by transformation from an exponential random variable. Derive the characteristic function of a Weibull distribution. Hence or otherwise determine the first four raw moments of this distribution.

UNIT – III

V. a) Show that for the Pearsonism distribution obtained from the equation

$$\frac{df(x)}{dx} = \frac{x\, f(x)}{b_0 + b_1 x + b_2 x^2}$$

the range is determined by the nature of the roots of the line quadratic equation $b_0 + b_1 x + b_2 x^2 = 0$. Under what conditions can this reduce to gamma distribution ?

b) What are orthogonal polynomials ? Explain the method of fitting an orthogonal polynomial.

VI. a) Explain the mathematical forms of Burr family of distributions and state all its properties.

b) Prove or disprove

i) Beta distribution, and

ii) Logistic distribution

are both members of Burr family of distribution.

UNIT – IV

VII. a) State and prove Fisher-Cochran theorem.

b) Let $X_1, X_2, \dots X_n$ be a random sample of size n drawn from $N(\mu, \sigma^2)$. Let S^2 be the sample variance. Derive the distribution of S^2 and hence obtain the first four moments of S^2.

VIII. a) Define non-central t statistic. Derive the probability distribution of this statistic.

b) Define non-central F distribution and explain an application of non-central F distribution in detail with an illustration.

UNIT – V

IX. a) Let $X_1, X_2, \dots X_n$ be a random sample of size n from a distribution, $f(x, \theta)$. Let $m_r = \frac{1}{n} \Sigma (x_i - \bar{x})^r$ be the rth sample moment about sample mean. Derive the large sample variance of m_r.

b) If $X_1, X_2, \dots X_n$ are i.i.d. random variables, show that $X_{(1)}$ is distributed as X_1 if and only if X_1 follows exponential law.

X. a) Derive the asymptotic distribution of sample median.

b) If $X_{1;n}, \dots X_{n:n}$ are the order statistics of a random sample of size n drawn from the standard exponential distribution and

$\mu_{r:n}^{(k)} = E(X_{r:n}^k)$; $k = 0, 1, \dots$; $1 \le r \le n$, then establish

$$\mu_{1:n}^{(k)} = \frac{k}{n} \cdot \mu_{1:n}^{(k-1)} \; ; \; k = 1, 2, \dots$$
