M 24422

Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com./M.Sc. Comp. Science Degree (Reg./Sup./Imp.) Examination, November 2013
STATISTICS
Paper – 1.2 : Analysis

Time: 3 Hours Max. Marks: 60

Instructions: *1) Answer* ***any five*** *question without omitting* ***any Unit.***
2) ***All*** *questions carry* ***equal*** *marks.*

UNIT – 1

I. a) Show that the sequence $\{S_n\}$ where

$$S_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n},\ \forall\, n \in N \text{ is convergent.}$$ 3

b) Define a closed set. Show that a set F is closed if and only if its complement is open. 5

c) Differentiate between limit point of a set and limit of a sequence. 4

II. a) Define a Cauchy sequence. Show that every convergent sequence is a Cauchy sequence. 4

b) State and prove Weierstrass's M-test for uniform convergence. 4

c) Test for convergence of the series

$$\frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} + \dots - \frac{1}{2} \le x \le \tfrac{1}{2}.$$ 4

UNIT – 2

III. a) Show that the function

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \ne (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

is continuous at the origin. 3

P.T.O.

b) Prove that the function $f(x, y) = \sqrt{|xy|}$ is not differentiable at the point (0, 0). 4

c) State and prove Roll's theorem. 5

IV. a) Investigate the maxima and minima of the function

$21x - 12x^2 - 2y^2 + x^3 + xy^2$. 3

b) Show that the function

$f(x, y, z) = 8z + 2x^2 + 3y^2 + 4z^2 - 3xy$ have a minimum at (0, 0, –1). 3

c) Prove that the volume of the greatest rectangular parallelopiped, that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $\frac{8abc}{3\sqrt{3}}$. 6

UNIT – 3

V. a) If $f_1 \in IR(\alpha)$ and $f_2 \in IR(\alpha)$ over [a, b], show that $f_1 + f_2 \in IR(\alpha)$ and

$$\int_a^b (f_1 + f_2)\, d\alpha = \int_a^b f_1\, d\alpha + \int_a^b f_2\, d\alpha.$$ 6

b) If f is monotonic on [a, b], and if α is continuous on [a, b] then show that $f \in IR(\alpha)$ on [a, b]. 3

c) Show that

$$\int_0^3 x^2\, (d[x] - x) = 5.$$

where [x] is the greatest integer not exceeding x. 3

VI. a) Evaluate the Euler Poisson integral

$$\int_0^\infty e^{-x^2}\, dx.$$ 4

b) Find the area of the domain, enclosed by y = x, y = 5x, x = 1. 4

c) Evaluate the surface integral

$$\iint_S \left(x^3\, dy\, dz + y^3\, dz\, dx + z^3\, dx\, dy\right)$$

over the sphere $x^2 + y^2 + z^2 = a^2$. 4

UNIT – 4

VII. a) If f(z) is analytic at z_0, prove that it is continuous at z_0. Is the converse true. **4**

b) Determine whether the function $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic. If so find V such that u + iV is harmonic. **4**

c) Show that the function $x^2 + iy^3$ is not analytic. **4**

VIII. a) If f(z) is analytic inside and on the boundary C of a simply connected region R, Prove that

$$f^1(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z-a)^2}\,dz.$$ **5**

b) Evaluate $\int_C \frac{e^{iz}}{z^3}\,dz$. where C is the circle $|z| = 2$. **3**

c) Show that $\frac{1}{2\pi i}\int_C \frac{e^{zt}}{z^2+1}\,dz = \sin t$, if t > 0 and C is the circle $|z| = 3$. **4**

UNIT – 5

IX. a) Expand f(z) = sin2 in a Taylor series about $z = \pi/4$. Also determine the region of convergence of this series. **4**

b) Find Laurent series about the indicated singularity in each case and give the region of convergence of each series.

i) $\frac{e^{2z}}{(z-1)^3}$; z = 1

ii) $(z-3)\sin\frac{1}{z+2}$; z = – 2. **8**

X. a) Show that $\int_0^\infty \frac{\sin x}{x}\,dx = \pi/2$. **4**

b) Prove that $\int_0^\infty \frac{\cos mx}{x^2+1}\,dx = \frac{\pi}{2}e^{-m}, m > 0$. **4**

c) Evaluate $\int_{-\infty}^\infty \frac{x \sin \pi x}{x^2+2x+5}\,dx$. **4**
