M 22388

Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com. Degree (Reg./Supple./Imp.) Examination, November 2012
STATISTICS
Paper – 1.2 : Analysis

Time : 3 Hours Max. Marks : 60

***Instructions**: 1) Answer **any five** questions without omitting **any** Unit.*
*2) **All** questions carry **equal** marks.*

UNIT – 1

I. a) Define open set show that every open set is a union of open intervals. 4

b) State and prove Bolzano-Weirstrass theorem. 4

c) Show that a set is closed if and only if its complement is open. 4

II. a) Show that every convergent sequence is bounded. 3

b) If $a_n = \sin\frac{n\pi}{2} + \frac{(-1)^n}{n}$, $n \in N$, then show that $\underline{\lim}\, a_n = -1$, $\overline{\lim}\, a_n = 1$. 3

c) If $\{f_n\}$ is a sequence of continuous functions on an interval [a, b] and if $f_n \to f$ uniformly on [a, b], then show that f is continuous on [a, b] 6

UNIT – 2

III. a) State and prove Legrange's mean value theorem. 4

b) Examine the function $(x-3)^5 (x+1)^4$ for extreme values. 4

c) Let $f(x) = \begin{cases} x^2 \sin 1/x, & x \neq 0 \\ 0 & , x = 0 \end{cases}$.

Prove that f(x) has a derivative at x = 0 and that f(x) and f′(x) are continuous at x = 0. 4

P.T.O.

IV. a) Find the maxima and minima of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. 3

b) Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225$. 5

c) If xyz = abc, show that minimum value of bcx + cay + abz is 3abc. 4

UNIT – 3

V. a) Define Rieman-Stieltjes integral. State and prove a necessary and sufficient condition for a function $f \in R(\alpha)$ on [a, b]. 5

b) If f is continuous on [a, b] show that $f \in R(\alpha)$ on [a, b]. 4

c) Evaluate $\int_0^2 x d\alpha(x)$ where $\alpha(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 + x, & 1 < x \le 2 \end{cases}$. 3

VI. a) Evaluate $\iint y\,dx\,dy$ over the part of the plane bounded by lines y = x and the parabola $y = 4x - x^2$. 4

b) Evaluate $\iint x^3 y^2 dx\,dy$ over the circle $x^2 + y^2 \le a^2$. 4

c) Find the Laplace transform of

i) $\sin^2 x$ ii) x sin wx 4

UNIT – 4

VII. a) If f(z) = u + iv is analytic in a region R prove that u and v are harmonic in R if they have continuous second partial derivatives in R. 4

b) Prove that $u = e^{-x}(x \sin y - y \cos y)$ is harmonic. Also find v such that f(z) = u + iv is analytic. 4

c) Verify that the Cauchy-Rieman equations are satisfied by the functions.

i) e^{z^2} ii) cos 2z. 4

VIII. a) State and prove Cauchy's integral formula. 5

b) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle |z| = 3. 3

c) If f is analytic inside and on a circle C of radious r and centre at z = a, prove that $\left|f^{(n)}(a)\right| \le \frac{\mu . n!}{r^n}$. Where μ is a constant such that $|f(z)| \le \mu$. 4

UNIT – 5

IX. a) State and prove Taylor's theorem. 6

b) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series valid for

i) $| < |z| < 3$ ii) $|z| > 3$ 6

X. a) Determine and classify all singularities of the function.

i) $\frac{1}{(2\sin z - 1)^2}$ ii) $\frac{z}{(e^x - 1)}$. 4

b) Show that $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta = \pi/12$. 4

c) Show that $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)^2 (x^2+2x+2)} = \frac{7\pi}{50}$. 4
