M 20483

Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, November 2011
STATISTICS
Paper – 1.2 : Analysis

Time: 3 Hours   Max. Marks: 60

Instructions: *1) Answer* ***any five*** *questions without omitting* ***any*** *Unit.*
2) ***All*** *questions carry* ***equal*** *marks.*

UNIT – 1

I. a) Show that every open interval is an open set. **3**

b) State and prove Hein-Borel theorem. **6**

c) Show that set of rational numbers is countable. **3**

II. a) If $\{a_n\}$ and $\{b_n\}$ are two sequences such that

i) $a_n \leq b_n$,

ii) $\lim a_n = a$ and $\lim b_n = b$ show that $a \leq b$. **4**

b) Show that

$$\lim_{n \to \infty} \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots + \frac{1}{\sqrt{n^2+n}} \right] = 1.$$ **4**

c) Prove that the sequence $\{f_n\}$ where $f_n(x) = \frac{x}{1+nx^2}$, x real converges uniformly on any closed interval I. **4**

UNIT – 2

III. a) Examine the validity of the hypothesis and the conclusion of Legrange's mean value theorem for

i) $f(x) = |x|$ on $[-1, 1]$

ii) $f(x) = \log x$ on $\left[\frac{1}{2}, 2\right]$. **4**

P.T.O.

b) Assuming the validity of expansion, show that

i) $e^x \cos x = 1 + x - \frac{2x^3}{3!} - \frac{2^2 x^4}{4!} ....$

ii) $\log \sec x = \frac{1}{2}x^2 + \frac{1}{12}x^4 + ...$ 4

c) If $2x + 3y + 4z = a$, show that maximum value of $x^2y^3z^4$ is $\left(\frac{a}{9}\right)^9$. 4

IV. a) Find the maxima and minima of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. 4

b) Show that the points on the ellipse $5x^2 - 6xy + 5y^2 = 4$ for which the tangent is at greatest distance from the origin are (1, 1) and (–1, –1). 4

c) Prove that a necessary condition for f(x, y) to have extreme value at (a, b) is that $f_x(a, b) = 0$, $f_y(a, b) = 0$, provided partial derivative exists. 4

UNIT – 3

V. a) If $f \in R(\alpha)$ over [a, b], then show that $|f| \in R(\alpha)$ and $\left|\int_a^b f d\alpha\right| \le \int_a^b |f| d\alpha$. 4

b) If $f \in IR(\alpha_1)$ and $f \in IR(\alpha_2)$ then show that $f \in R(\alpha_1 + \alpha_2)$ and

$\int_a^b fd(\alpha_1 + \alpha_2) = \int_a^b fd\alpha_1 + \int_a^b fd\alpha_2$. 4

c) Evaluate :

i) $\int_0^2 [x]\, dx^2$, where [x] is integer part of x.

ii) $\int_0^3 f(x)\, d([x] + x)$ where $f(x) = |x|, 0 \le x < 3/2$.

$= e^x, 3/2 \le x \le 3$. 4

VI. a) Show that $\iint_E \sqrt{xy}\, dx\, dy = \pi/24$ where E is the region bounded by the lines $x = 0, y = 0, x + y - 1 = 0.$ **4**

b) Evaluate :

$$\iint_E \frac{y+1}{x^2 + (y+1)^2} dx\, dy$$

where $E = \{(x, y)/y \geq 0, x^2 + y^2 \leq 1\}.$ **4**

c) Find the area of the surface of the cylinder $x^2 + y^2 = 4a^2$ above the xy – plane and bounded by the planes $y = 0, z = a$ and $y = z$. **4**

UNIT – 4

VII. a) Prove that a necessary condition that $w = u + iV$ be analytic in a region R is that the Cauchy-Riemann equation $u_x = V_y$ and $u_y = -V_x$ are satisfied. **3**

b) Prove that the function $u = 2x(1 - y)$ is harmonic. Also find the function V such that $f(z) = u + iV$ is analytic. **5**

c) Prove that if the function $w = f(z) = u + iV$ is analytic in a region R, then

$$\frac{dw}{dz} = \frac{dw}{dx} = -i\frac{dw}{dy}.$$ **4**

VIII. a) If f(z) is analytic in a region R prove that $f'(2), f''(2), \ldots$ are analytic in R. **4**

b) Find the value of

i) $\int_C \frac{\sin^6 z}{z - \pi/6} dz$

ii) $\int_C \frac{\sin^6 z}{(z - \pi/6)^3} dz$ if C is the circle $|z| = 1$. **8**

UNIT – 5

IX. a) Show that $\tan^{-1} z = z - \frac{z^3}{3} + \frac{z^5}{5} - \frac{z^7}{7} + ..., |z| < 1.$ 4

b) Expand $f(z) = \frac{1}{z(z-2)}$ in a Laurent series valid for

i) $0 < |z| < 2$

ii) $|z| > 2.$ 8

X. a) Show that $\int_0^{2\pi} \frac{d\theta}{(5 - 3\sin\theta)^2} d\theta = \frac{5\pi}{32}$ 4

b) Evaluate $\int_{-\infty}^{\infty} \frac{x \sin \pi x}{x^2 + 2x + 5} dx.$ 4

c) Show that $\int_0^{\infty} \frac{x^{P-1}}{1+x} dx = \frac{\pi}{\sin P\pi}$ 4
