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M 16932

Reg. No. : A8PSST1005

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First Semester M.Sc. Degree Examination, November 2009
STATISTICS
Paper – 1.2 : Analysis

Time: 3 Hours　　　　　　　　　　　　　　　　　　Max. Marks: 60

*Instructions : 1) Answer **any five** questions without omitting **any** Unit.*
*2) **All** questions carry **equal** marks.*

UNIT – 1

I. a) Define :

i) Open set　　　　　　ii) Closed set

Show that the set $S=\left\{1,-1,1\frac{1}{2},-1\frac{1}{2},1\frac{1}{3},-1\frac{1}{3}..\right\}$ is closed but not open.

b) Prove : A set is closed if and only if its complement is open.

c) Define convergence of a sequence. Show that $\{a_n\}$ where $a_n=\left(1+\frac{1}{n}\right)^n$ is convergent.

II. a) State and prove Hein-Borel theorem.

b) Define continuous functions. Prove : If a function is continuous in a closed interval, then it is bounded therein.

c) Show that the series $\sum_{n=2}^{\alpha}\frac{1}{n(\log n)^p}, p>0$ converges for $p>1$ and diverges for $p\leq 1$.

P.T.O.

UNIT – 2

III. a) Prove or disprove : A function which is differentiable at a point is necessarily continuous at that point.

b) State and prove Rolle's theorem.

c) Show that sin x (1 + cos x) is maximum at $x = \pi/3$.

IV. a) Evaluate :

i) $\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x}$ ii) $\lim_{x \to 0} \frac{x - \sin x}{x^2}$.

b) Define maxima and minima of a function of two variables. Show that $f(x, y) = y^2 + x^2y + x^4$ has a minimum at (0, 0).

c) If xyz = abc, show that the minimum value of bcx + cay + abz is 3abc.

UNIT – 3

V. a) Define Rieman–Stieltjes integral. Find $\int_{-1}^{1} x\, d[x]$ where [x] is the greatest integer less than or equal to x.

b) Prove or disprove : If $f \in R(\alpha)$ on [a, b], then $|f| \in R(\alpha)$ on [a, b].

c) Every finite sum can be expressed as Rieman–Stieltjes integral – Prove.

VI. a) Let $F(x) = \int_0^x f(t)dt$. Under certain conditions on f, to be stated, show that

$F'(x) = f(x)$, where $F'(x) = \frac{dF(x)}{dx}$

b) Evaluate $\iiint_D \frac{x_1 x_2 x_3}{3} dx_1 dx_2 dx_3$ where D is a sphere of radius r with centre origin.

c) Find the Laplace transform of :

i) $t^2 \sin hat$ ii) $\int_0^t \frac{\tan x}{x} dx$.

UNIT – 4

VII. a) Prove or disprove : All continuous functions are analytic.

b) Establish Cauchy-Riemann equations for analytic functions.

c) Define harmonic function. Find the harmonic conjugate of $u = x^2 - y^2 - 4xy - x + y$.

VIII. a) State and prove Cauchy's fundamental theorem.

b) Evaluate $\int_c \frac{1}{z^2} dz$ around the circle $|z| = 1$.

c) State Cauchy's integral formula. Find $\int_C \frac{e^z \sin z}{z^2} dz$ where C is the circle $|z| = 2$.

UNIT – 5

IX. a) State and prove Taylor's series theorem.

b) Obtain Maclaurin series expansion for $\frac{1}{1+z}$ for $|z| < 2$.

c) Define :

i) Residue and ii) Pole

Obtain residues of $f(z) = \frac{e^z}{z^2 + 4}$ at all its poles.

X. a) State and prove Cauchy's residue theorem.

b) Expand cos z into a Taylor series about $z = \frac{\pi}{2}$.

c) Evaluate :

i) $\int_0^{\pi} \frac{\sin\theta}{a + b\cos\theta} d\theta$ ii) $\int_0^{\infty} \frac{\sin x}{x} dx$.
