M 15468

Reg. No. : 08PSST1003

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First Semester M.Sc. Degree Examination, December 2008
Paper – 1.2 : STATISTICS
Analysis

Time : 3 Hours

Max. Marks : 60

Instructions : Answer any five questions without omitting any Unit. All questions carry equal marks.

UNIT – 1

I. a) Define closed set. Prove or disprove the union of infinitely many closed sets is closed.

b) Show that the set $S = \left\{1, -1, \frac{1}{2}, \frac{-1}{2}, \frac{1}{3}, \frac{-1}{3} \ldots\ldots\right\}$ is neither open nor closed.

c) Prove : Every bounded infinite set has the smallest and greatest limit points.

II. a) State Cauchy's principle of convergence. Show the sequence $\left\{\frac{(-1)^n}{n}\right\}$ is convergent.

b) Prove : Every convergent sequence is bounded.

c) Examine the continuity at x = 0 if

i) $f(x) = \begin{cases} \frac{\tan^{-1} x}{x}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$

ii) $f(x) = \begin{cases} x, & \text{when x is rational} \\ 0, & \text{when x is irrational} \end{cases}$.

P.T.O.

UNIT – 2

III. a) If f(x) and g(x) are differentiable at x = c, show that the functions f(x) + g(x) and f(x)g(x) are also differentiable at x = c.

b) State Lagrange's mean value theorem. Using this show that

$$0 < \frac{1}{\log(1-x)} - \frac{1}{x} < 1, \text{ for all } x > 0.$$

c) State Taylor's theorem. Show that the maximum value of

$\frac{\log x}{x}$ is $\frac{1}{e}$, for $0 < x < \infty$.

IV. a) Evaluate :

i) $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$

ii) $\lim_{x\to \frac{\pi}{2}}(\sec x - \tan x)$.

b) Show that $f(x, y) = (y - x)^4 + (x - 2)^4$ has a minimum at (2, 2).

c) Find the shortest distance between the point P and Q when P moves on the plane x + y + z = 2a and Q on the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

UNIT – 3

V. a) Define Rieman-Stieltjes integral. If $f \in R(\alpha)$ on [a, b], show that $f^2 \in R(\alpha)$ on [a, b].

b) Prove or disprove :

If f is continuous on [a, b], then $f \in R(\alpha)$, where α is any real valued function.

c) Let f be a function bounded on [–1, 1]. Let $\beta_1(x) = 0$, if $x \le 0$ and 1, if $x > 0$. Show that $f \in R(\beta)$ if and only if f is continuous at x = 0.

VI. a) State and prove first mean value theorem on Rieman-Stieltjes integral.

b) Evaluate $\iint_D x^2 dxdy$, where D is a circle of radius one centre at the origin.

c) Find Laplace transform of

i) $e^{bt} \sin at$

ii) $\int_0^t \frac{\cos x}{x} dx$.

UNIT – 4

VII. a) Define analytic function. Obtain a necessary and sufficient condition for analyticity of a function f(z).

b) Let f(z) = u(x, y) + iv(x, y). If f(z) is analytic, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$.

c) Define harmonic function. Prove or disprove $u(x, y) = x^2 - y^2$ is harmonic.

VIII. a) State Cauchy fundamental theorem. Evaluate $\int_C \frac{e^z}{z^3} dz$, around the circle |z| = 1.

b) State and prove Cauchy's integral formula.

c) Evaluate $\int_C \frac{\sin z}{(z-2)^2} dz$, where C is the circle |z – 2| = 1.

UNIT – 5

IX. a) State and prove Laurent series theorem.

b) Obtain Maclaurin series expansion for $f(z) = \frac{1}{1+z^2}$.

c) Define :

i) Residue

ii) Pole and

iii) Zero.

Find the residues of $f(z) = \dfrac{\cot z \coth z}{z^3}$ at $z = 0$.

X. a) State Cauchy residue theorem. Using this find $\int_C \dfrac{e^{xz}}{z^2(z^2+2z+2)} dz$ around the circle C with equation $|z| = 3$.

b) Expand sin z in a Taylor series about $z = \pi/4$.

c) Evaluate :

i) $\displaystyle\int_{-\infty}^{\infty} \frac{x \sin \pi x}{x^2+2x+5} dx$

ii) $\displaystyle\int_{0}^{\infty} \frac{\log(1+x^2)}{1+x^2} dx$.

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