Reg. No. : ....

Name : ....

M 13934

First Semester M.Sc. Degree Examination, November 2007
STATISTICS
Paper – 1.2 : Analysis

Time: 3 Hours Max. Marks: 60

***Instructions :** Answer **any five** questions **without** omitting **any** Unit. All questions carry **equal** marks.*

UNIT – 1

1. a) Define closed set. Prove that every closed set is the intersection of a countable collection of open sets.

   b) Prove or disprove, Every bounded sequence has a limit point.

   c) Consider the sequence $|a_n|$ where $a_n = \sqrt{n+1} - \sqrt{n}$. Show that the sequence converges to zero.

2. a) Establish Cauchy's principle of convergence of sequence of real numbers.

   b) Suppose that $f : D \to R$ is continuous and that D is closed and bounded. Show that f(x) is bounded in D.

   c) Let $\sum_{1}^{\infty} f_n(x)$ be a series of functions defined on DCR. If there exists a sequence $\{M_n\}_{n=1}^{\infty}$ of constants, such that $|f_n(x)| \leq M_n$ n = 1, 2 ... for all $x \in D$ and if $\sum_{1}^{\infty} M_n$ converges, show that $\sum_{n=1}^{\infty} f_n(x)$ converges uniformly on D.

UNIT – 2

3. a) Discuss the differentiability of the following function at x = 0

   i) $f(x) = x\,|\,x\,|$ ii) $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

P.T.O.

b) State and prove Lagrange's mean value theorem.

c) State Taylor's theorem. Show that $\log \sec x = \frac{x^2}{2} + \frac{x^4}{12} + \ldots\ldots\ldots$

4. a) Show that the function $(y - x)^4 + (x - 2)^4$ has a minimum at (2, 2).

b) If f(x) is monotone increasing on (a, b) and if f(x) is differentiable on (a, b), then show that $f'(x) \geq 0$ on (a, b).

c) Evaluate :

i) $\lim_{x \to 0} \frac{x - \tan x}{x^2}$

ii) $\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$.

UNIT – 3

5. a) Define Reiman-Stieltjes integral. Let $f(x) = \alpha(x) = 0$ for $a \leq x < c$; $f(x) = \alpha(x) = 1$ for $c < x \leq b$, $f(c) = 0$ and $\alpha(x) = 1$. Prove or disprove $\int_a^b f(x)\, d\alpha(x) = 0$.

b) Suppose that f(x) is Rieman-Stieltjes integrable with respect to $\alpha(x)$ on [a, b]; where $\alpha(x)$ has a continuous derivative $\alpha'(x)$ on $[a, b]$. Show that $\int_a^b f(x)\, d\alpha(x) = \int_a^b f(x)\, \alpha'(x)\, dx$.

c) If f is $R(\alpha)$ and g is $R(\alpha)$ on [a, b], show that the product f.g is $R(\alpha)$ on [a, b].

6. a) State and prove Rieman's condition for the existence of Rieman-Stieltjes integral.

b) Prove or disprove : If f(x) is continuous on [a, b] and $\alpha(x)$ is monotonic on [a, b], then f(x) is Rieman integrable with respect to $\alpha(x)$ on [a, b].

c) Using Laplace transform, evaluate

i) $\int_0^\infty \frac{\sin t}{t^2} dt$ and

ii) $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} dt$

UNIT – 4

7. a) Define analytic function. Show that $f(z) = x^2 + iy^2$ is not analytic anywhere.

b) If f(z) is an analytic function on C, show that $\int_c f(z)dz = 0$.

c) Evaluate $\int_c \frac{dz}{z-1}$ around (i) the circle $|z - 2| = 4$ and (b) the square with vertices at $2 \pm 2i$ and $-2 \pm 2i$.

8. a) Show that $u = e^{-x}(x \sin y - y \cos y)$ is harmonic and find its conjugate.

b) State and prove Morera's theorem.

c) State Cauchy's integral formula. Evaluate $\frac{1}{2\pi i} \int_C \frac{e^z}{z-\alpha} dz$ where C is the circle with $|z| = 1$.

UNIT – 5

9. a) State and prove Taylor's series theorem.

b) By Laurent series expansion, show that $z^2 e^{-z^4} = z^2 - z^6 + z^{10}/2! - \frac{z^{14}}{3!} + ...$

c) State Cauchy residual theorem. Evaluate the $\int_c \frac{e^z dz}{\cos echz}$ around the circle C defined by $|z| = 5$.

10. a) Obtain Maclaurin series expansion for $\frac{1}{1+z^2}$ for $|z|<1$.

b) Define :

i) Singularity

ii) Poles and residue of $f(z) = \frac{\cos tz \coth z}{z^3}$ at $z = 0$

c) Evaluate :

i) $\int_0^\infty \frac{\cos 2\pi x}{x^4 + x^2 + 1} dx$ and

ii) $\int_0^{2\pi} \frac{\sin 3\theta}{5 - 3\cos\theta} d\theta$
