M 10987

I Sem. M.Sc. Degree Examination, November 2005
Paper – 1.2: STATISTICS
Analysis

Time: 3 Hours Max. Marks: 60

Instructions: *1) Answer any five questions **without** omitting any Unit.*
*2) All questions carry **equal** marks.*

UNIT – 1

1. a) Define limit point of a set. Show that a set is closed if and only if its complement is open.

   b) Show that every convergent sequence is bounded and has a unique limit.

   c) Use Cauchy's principle of convergence to show that $\left\{\frac{(-1)^n}{n}\right\}$ is convergent.

2. a) State and prove Hein-Borel theorem.

   b) Show that a necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded.

   c) Examine the continuity at x = 0 if

   i) $f(x) = x - |x|$

   ii) $f(x) = \frac{\tan^{-1} x}{x}$ if $x \neq 0$

   $= 0$ if $x = 0$

UNIT – 2

3. a) Discuss the differentiability of the functions at x = 1

   i) $f(x) = \begin{cases} 2 & x \leq 1 \\ x & x > 1 \end{cases}$

   ii) $f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ x & x > 1 \end{cases}$

P.T.O.

b) State and prove Rolles theorem.

c) Suppose that the function defined on [a, b] is continuous and differentiable. Show that there exists at least one real number $c \in [a, b]$ such that $f(b) - f(a) = (b - a) f'(c)$.

4. a) State and prove Taylor's theorem.

b) Show that Sin x (1 + cos x) is maximum at $x = \frac{\pi}{3}$

c) Evaluate

i) $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$

ii) $\lim_{x \to 0} (1 - x)^x$.

UNIT – 3

5. a) If $f \in R(\alpha)$ on [a, b], then show that α is Riemann integral with respect to f on [a, b] and

$$\int_a^b f(x)\, d\alpha(x) + \int_a^b \alpha(x)\, df(x) = f(b)\alpha(b) - f(a)\alpha(a)$$

b) If $f \in R(\alpha)$ on [a, b] and if $\int_a^b f\, d\alpha = 0$ for every f which is monotonic on [a, b], show that α must be constant on [a, b].

c) Show that $\int_0^3 x\, d([x] - x) = \frac{3}{2}$ where [x] is the greatest integer not exceeding x.

6. a) If f is $R(\alpha)$ on [a, b] show that $|f|$ is $R(\alpha)$ on [a, b].

b) State and prove first mean value theorem on Riemann Stieltjes integral.

c) Define Laplace transform. Obtain the Laplace transform of the n^{th} derivative of a function.

UNIT – 4

7. a) Define analytic function. Find $f'(z)$ when $f(z) = \frac{(z-1)}{(2z+1)}$; $z \neq \frac{-1}{2}$

b) Derive necessary and sufficient condition for analyticity of a function.

c) Derive Cauchy integral formula.

8. a) Define harmonic function. Show that $u(x, y) = 2x(1 - y)$ is harmonic and find a harmonic conjugate $v(x, y)$.

b) Evaluate $\int_C f(z)\,dz$ when $f(z) = \frac{z+2}{z}$ and C is the semicircle $z = 2e^{i\theta}$ $(0 \leq \theta \leq M)$.

c) State and prove Cauchy fundamental theorem.

UNIT – 5

9. a) State Taylor's series theorem. Expand cos z into a Taylor series about the point $z = \pi/2$.

b) By Laurent series representation, show that $\frac{e^z}{z^2} - \frac{1}{z^2} + \frac{1}{z} + \frac{1}{2!} + \frac{z}{3!} + ...$

c) State and prove Cauchy residue theorem.

10. a) Obtain Maclaurin series expansion for $\frac{1}{1-z}$.

b) Define poles and residues. For $f(z) = \frac{z}{\cos z}$, Show that the singular points are poles.

c) Evaluate :

i) $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)^2}$ ii) $\int_{-\infty}^{\infty} \frac{\cos x\,dx}{(x+a)^2+b^2}$

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