M 770

Reg. No. : .....

Name : .....

IV Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal Ul Ulama Degree (CCSS – Reg./Supple./Improv.) Examination, April 2012

COMPLEMENTARY COURSE IN MATHEMATICS

4C04 MAT : Numerical Analysis and Vector Calculus

Time: 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) The angle between $4\bar{i} + 3\bar{k}$ and $\bar{j}$ is _____

   b) IF $\bar{a} = (\cos x)y^2\bar{i} + (y + y^3 \sin y)\bar{j} + \cos^3 y\bar{k}$, then $\frac{\partial^2 \bar{a}}{\partial y^2}$ = _____

   c) $2\bar{i} + \bar{j} + \bar{k}$ and _____ are collinear vectors.

   d) Relation between a × b and b × a is _____ **(Wt. 1)**

Answer **any 6** from the following (Weightage **1 each**) :

2. Use Newton's method to find a root of the equation $x^3 - 3x - 5 = 0$. Start with $x_0 = 3$

3. Construct forward difference table from the following values of x and y :

x :	3.0	3.1	3.2	3.3	3.4
y :(= 1/x)	0.33333	0.32258	0.31250	0.30303	0.29412

4. Solve by Gauss – Elimination

$$x_1 + x_2 + x_3 = 3$$
$$2x_1 + 3x_2 + x_3 = 6$$
$$x_1 - x_2 - x_3 = -3$$

5. Derive an interpolation formula for equal intervals.

P.T.O.

6. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$, with y = 1 for x = 0. Find y approximately for x = 0.1 by Euler's method.

7. Given the 3 vectors $\bar{A} = 3\bar{i} + 2\bar{j} + 6\bar{k}$, $\bar{B} = 3\bar{i} + 4\bar{k}$, $\bar{C} = 2\bar{i} - 2\bar{j} + \bar{k}$, evaluate $\bar{A} \times \bar{B} \cdot \bar{C}$.

8. Find the arc length between (0,0,1) and (1, 0, 1) of the helix $y = \sin 2\pi x$, $z = \cos 2\pi x$.

9. The position vector of a moving particle is $R = \cos t\,(\bar{i} - \bar{j}) + \sin t(i + j) + \frac{1}{2} t\,\bar{k}$. Find a unit tangent to the path of the particle, in the direction of motion.

10. Find the directional derivative df/ds at (1, 3, –2) in the direction of $-\bar{i} + 2\bar{j} + 2\bar{k}$ if f(x, y, z) = yz + xy + xz. (6×1=6)

Answer **any 7** from the following (Weightage **2 each**) :

11. Given $\log_{10} 654 = 2.8156$, $\log_{10} 658 = 2.8182$, $\log_{10} 659 = 2.8189$, $\log_{10} 661 = 2.8202$; find $\log_{10} 656$.

12. Calculate by Trapezoidal rule an approximate value of $\int_{-3}^{3} x^4 dx$ by taking 7 equidistant ordinates. Compare it with exact value.

13. Solve by Gauss – Jordan Reduction method :

$10x_1 + x_2 + x_3 = 12$

$x_1 + 10x_2 + x_3 = 12$

$x_1 + x_2 + 10x_3 = 12$

14. Find the inverse of $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ by Gauss – Elimination method.

15. Given $y^1 = dy/dx = y^2 + 1$ with boundary condition y = 0 at x = 0 in the range $0 \le x \le 1$, obtain y as a series in powers of x.

16. a) Find div f, given that $F = x\bar{i} + y\bar{j} + z\bar{k}$.

    b) Give an example of a field with a constant negative divergence.

17. a) If $F = xy^2\,\bar{i} + xy\,\bar{j} + xy\,\bar{k}$, find curl F.

    b) Can you find a vector field whose curl is $y\,\bar{i}$?

18. a) If $F(x, y, z) = x^2y\,\bar{i} + z\bar{j} - (x + y - z)\,\bar{k}$, find $\nabla(\nabla \cdot F)$.

    b) Find $\nabla \cdot R$ where $R = x\bar{i} + y\bar{j} + z\bar{k}$.

19. Show that $\nabla \cdot (F \times G) = G \cdot (\nabla \times F) - F \cdot (\nabla \times G)$ where F and G are vector fields.

20. Evaluate $\int_C e^{xyz}(yzdx + xzdy + xydz)$ where C is the straight-line segment joining the point (2, 1, 4) to the point (3, 3, 4). **(7×2=14)**

Answer **any three** from the following (Weightage **3 each**) :

21. Use Runge-Kutta fourth order method to solve $y' = xy$ for $x = 1.4$. Initially $x = 1$, $y = 2$. (take $h = 0.2$).

22. Find the value of $f'(0.04)$ from the following table :

x	:	.01	.02	.03	.04	.05	.06
y=f(x)	:	.1023	.1047	.1071	.1096	.1122	.1148

23. Show that $F = 2xyi + (x^2 + 1)\,j + 6z^2\bar{k}$ is conservative, and find a scalar potential ϕ.

24. Compute $\iint F \cdot d\,S$ where S is the surface of the cube bounded by the planes $x = \pm 1$, $y = \pm 1$, $z = \pm 1$, if $F = x\bar{i} + y\bar{j}$.

25. Evaluate $\iint_S (\nabla \times F) \cdot d\,S$ where $F = y\bar{i} + (x - 2x^3z)\bar{j} + xy^3\bar{k}$ and S is the surface of a sphere $x^2 + y^2 + z^2 = a^2$ above the xy – plane. **(3×3=9)**
