K22U 3422

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2022
(2019 Admission Onwards)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
1C01 MAT – ST : Mathematics for Statistics – I

Time : 3 Hours   Max. Marks : 40

PART – A

Answer **any four** questions from among the questions **1** to **5**. **Each** question carries **one** mark.

1. Find the n^{th} derivative of $(ax + b)^m$.

2. State Rolle's theorem.

3. Evaluate $\lim\limits_{x \to 0} \dfrac{\log x}{\cot x}$.

4. Define rank of a matrix.

5. Find parametric equation for the line through P(–3, 2, –3) and Q(1, –1, 4).

PART – B

Answer **any seven** questions from among the questions **6** to **15**. **Each** question carries **2** marks.

6. If y = sin(sin x), prove that $\dfrac{d^2y}{dx^2} + \tan x \dfrac{dy}{dx} + y \cos^2 x = 0$.

7. If $y = (2 - 3x)^{10}$, find y_9.

P.T.O.

8. Verify Lagrange's mean value theorem for the function f(x) = (x – 1) (x – 2) (x – 3) in (0, 4).

9. Using Maclaurin's series, expand sin x.

10. Determine the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

11. Solve the equations 3x + y + 2z = 3, 2x – 3y – z = –3, x + 2y + z = 4 by determinants.

12. Show that the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular.

13. Find an equation for the plane through A(0, 0, 1), B(2, 0, 0) and C(0, 3, 0).

14. A glider is soaring upward along the helix r(t) = (cos t) i + (sin t) j + t k. How long is the glider's path from t = 0 to t = 2π ?

15. Find T and N for the circular motion r(t) = (cos2t) i + (sin2t) j.

PART – C

Answer **any four** questions from among the questions **16** to **22**. **Each** question carries **three** marks.

16. If $y = x \log \frac{x-1}{x+1}$, show that $y_n = (-1)^{n-2}(n-2)!\left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$.

17. Find the n^{th} derivative of $\frac{1}{x^2 - 6x + 8}$.

18. Verify Cauchy's mean value theorem for the functions e^x and e^{-x} in the interval (a, b).

19. Expand $\log_e x$ in powers of (x – 1) and hence evaluate $\log_e 1.1$ correct to 4 decimal places.

20. Evaluate $\lim\limits_{\theta \to 0} \frac{\theta - \sin\theta}{\sin\theta(1 - \cos\theta)}$.

21. Reduce the matrix $A = \begin{bmatrix} 1 & -1 & -2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ to its normal form and hence find its rank of A.

22. Test for consistency and solve $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$.

PART – D

Answer **any two** questions from among the questions **23** to **26**. **Each** question carries **five** marks.

23. If $y = (\sin^{-1}x)^2$, show that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - n^2y_n = 0$. Hence find $(y_n)_0$.

24. Prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$, where $0 < a < b < 1$. Hence deduce that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.

25. Solve the equations $x_1 - x_2 + x_3 + x_4 = 2$, $x_1 + x_2 - x_3 + x_4 = -4$, $x_1 + x_2 + x_3 - x_4 = 4$, $x_1 + x_2 + x_3 + x_4 = 0$ by finding inverse by elementary row operations.

26. a) Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $v = 2i - 3j + 6k$.

    b) In what directions does f change most rapidly at P_0 and what are the rates of change in these directions ?