Reg. No. : $\qquad$
Name: $\qquad$
I Semester B.Sc. Degree (C.B.C.S.S.- O.B.E. - Regular/Supplementary/ Improvement) Examination, November 2022
(2019 Admission Onwards) CORE COURSE IN MATHEMATICS
1B01 MAT : Set Theory, Differential Calculus and Numerical Methods
Time : 3 Hours
Max. Marks : 48
PART - A

Answer four questions from this Part. Each question carries one mark.

1. Define a relation on set of integers.
2. Find the $\mathrm{n}^{\text {th }}$ derivative of $\mathrm{x}^{3}+5 \mathrm{x}^{2}+3$.
3. Find the domain of $\log x$.
4. State Euler's theorem on homogeneous functions.
5. Let $w=x^{2} y+2 y$. Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$.
PART - B

Answer any eight questions from this Part. Each question carries two marks.
6. Define reflexive relation on a set and give an example.
7. Show that the relation $\leq$, is a partial relation on set of all real numbers.
8. Give an example of a function which is one-one, but not on-to.
9. Give an example of algebraic and transcendental equation.
10. Find $\lim _{x \rightarrow 0} f(x)$, where $f(x)=\left\{\begin{array}{ll}0 & \text { if } x \geq 0 \\ 1 & \text { if } x<0\end{array}\right.$.
11. Show that $\lim _{\theta \rightarrow 0} \sin \theta=0$.
12. For what values of $a$ is $f(x)=\left\{\begin{array}{c}a^{2} x-2 a \text {, if } x \geq 2 \\ 12 \text { if } x \leq 2\end{array}\right.$ continuous at every $x$ ?
13. Describe the domain of the function $f(x, y)=\sqrt{y-x^{2}}$.
14. Find the values of $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ at the point $(4,-5)$ if $f(x, y)=x^{2}+3 x y+y-1$.
15. Let $f(x, y)=2 x+3 y-4$. Find the slope of the line tangent to this surface at the point $(2,-1)$ and lying in the plane $x=2$.
16. Find $\frac{d y}{d x}$ using implicit differentiation, if $y^{2}-x^{2}-\sin x y=0$.
PART - C

Answer any four questions from this Part. Each question carries four marks.
17. Show that $(A \times B) \cap(A \times C)=A \times(B \cap C)$.
18. Let $A=\{a, b\}$ and $B=\{1,2,3\}$
i) Find number of functions from $A$ to $B$
ii) Find number of functions from $B$ to $A$.
19. Determine the maximum number of positive and negative roots and intervals of length one unit in which the real roots lie for the following equation $8 x^{3}-12 x^{2}-2 x+3=0$.
20. If $a x^{2}+2 h x y+b y^{2}=1$, then show that $\frac{d^{2} y}{d x^{2}}=\frac{h^{2}-a b}{(h x+b y)^{3}}$.
21. Evaluate $\lim _{x \rightarrow 0} \frac{\sqrt{x^{2}+100}-10}{x^{2}}$.
22. Describe the level surfaces of the function $f(x, y, z)=\sqrt{x^{2}+y^{2}+z^{2}}$.
23. State mixed partial theorem. Verify it for the function $w=x \sin y+y \sin x+x y$.

## PART - D

Answer any two questions from this Part. Each question carries six marks.
24. i) Let $f: R \rightarrow R$, defined by $f(x)=2 x-3$, find the formula for $f^{-1}$.
ii) Consider the function $\mathrm{f}: \mathrm{A} \rightarrow \mathrm{B}, \mathrm{g}: \mathrm{B} \rightarrow \mathrm{C}$, then prove that if g of is one-one, then $f$ is one-one.
25. Find the root correct to two decimal places of the equation $f(x)=x e^{x}-\cos x=0$, using the method of false position.
26. If $y^{\frac{1}{m}}+y^{-\frac{1}{m}}=2 x$, then show that $\left(x^{2}-1\right) y_{n+2}+(2 n+1) x y_{n+1}+\left(n^{2}-m^{2}\right) y_{n}=0$.
27. i) Find $\lim _{(x, y) \rightarrow(0,0)} \frac{x^{2}-x y}{\sqrt{x}-\sqrt{y}}$.
ii) At what points $(x, y)$ in the plane are the function $f(x, y)=\sin \frac{1}{x y}$ is continuous.

