

K22U 3419

Reg. No. :

Name :

I Semester B.Sc. Degree (C.B.C.S.S.– O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2022 (2019 Admission Onwards) CORE COURSE IN MATHEMATICS 1B01 MAT : Set Theory, Differential Calculus and Numerical Methods

PART – A

Time : 3 Hours

Max. Marks: 48

Answer four questions from this Part. Each question carries one mark.

- 1. Define a relation on set of integers.
- 2. Find the nth derivative of $x^3 + 5x^2 + 3$.
- 3. Find the domain of log x.
- 4. State Euler's theorem on homogeneous functions.
- 5. Let w = x²y + 2y. Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$.

PART – B

Answer any eight questions from this Part. Each question carries two marks.

- 6. Define reflexive relation on a set and give an example.
- 7. Show that the relation \leq , is a partial relation on set of all real numbers.
- 8. Give an example of a function which is one-one, but not on-to.
- 9. Give an example of algebraic and transcendental equation.
- 10. Find $\lim_{x \to 0} f(x)$, where $f(x) = \begin{cases} 0 & \text{if } x \ge 0 \\ 1 & \text{if } x < 0 \end{cases}$.

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11. Show that $\lim_{\theta \to 0} \sin \theta = 0$.

12. For what values of a is
$$f(x) = \begin{cases} a^2x - 2a, & \text{if } x \ge 2\\ 12 & \text{if } x \le 2 \end{cases}$$
 continuous at every x ?

13. Describe the domain of the function $f(x, y) = \sqrt{y - x^2}$.

- 14. Find the values of $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ at the point (4, -5) if f(x, y) = x² + 3xy + y 1.
- 15. Let f(x, y) = 2x + 3y 4. Find the slope of the line tangent to this surface at the point (2, -1) and lying in the plane x = 2.
- 16. Find $\frac{dy}{dx}$ using implicit differentiation, if $y^2 x^2 \sin xy = 0$.

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PART – C
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Answer any four questions from this Part. Each question carries four marks.

- 17. Show that $(A \times B) \cap (A \times C) = A \times (B \cap C)$.
- 18. Let $A = \{a, b\}$ and $B = \{1, 2, 3\}$
 - i) Find number of functions from A to B
 - ii) Find number of functions from B to A.
- 19. Determine the maximum number of positive and negative roots and intervals of length one unit in which the real roots lie for the following equation $8x^3 12x^2 2x + 3 = 0$.
- 20. If $ax^{2} + 2hxy + by^{2} = 1$, then show that $\frac{d^{2}y}{dx^{2}} = \frac{h^{2} ab}{(hx + by)^{3}}$.

21. Evaluate $\lim_{x\to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}$.

- 22. Describe the level surfaces of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.
- 23. State mixed partial theorem. Verify it for the function $w = x \sin y + y \sin x + xy$.

PART – D

Answer any two questions from this Part. Each question carries six marks.

- 24. i) Let $f : \mathbb{R} \to \mathbb{R}$, defined by f(x) = 2x 3, find the formula for f^{-1} .
 - ii) Consider the function $f : A \rightarrow B$, $g : B \rightarrow C$, then prove that if g of is one-one, then f is one-one.
- 25. Find the root correct to two decimal places of the equation $f(x) = xe^x \cos x = 0$, using the method of false position $\cos x = 0$

26. If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, then show that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

- 27. i) Find $\lim_{(x,y)\to(0,0)} \frac{x^2 xy}{\sqrt{x} \sqrt{y}}$.
 - ii) At what points (x, y) in the plane are the function $f(x, y) = \sin \frac{1}{xy}$ is continuous.

